UNDERSTANDING YEAR 11 & 12 GENERAL MATHS

KERRI FUNNELL

Five Senses Education Pty Ltd
ABN 16 001 414 437
2/195 Prospect Highway
Seven Hills 2147
New South Wales
Australia

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CHAPTER 1

THE CALCULATOR

THE IMPORTANCE OF THE CALCULATOR

Because the General course includes many topics which cover a more practical relationship between Mathematics and everyday living, the calculator obviously plays an extremely important role. Most students should, by year 11, be very familiar with their calculator, as it should have been used frequently in years 9 and 10 – if not earlier. Therefore it is assumed that you are by now confident in its basic operations (+ − × ÷), and only a brief description of the other important keys and their functions is included here.

> It is essential to be able to use your calculator with confidence – because most exam questions involve calculator work.

Most schools recommend their students purchase the *fx*-range of direct logic calculators. There are a variety of models in the *fx*-range, but most of these models have the same keys, although they might be in different positions, or have slight variations in their names.

The important point to note is that each key usually has two different functions (or jobs) to perform. This idea saves space and prevents the calculator from becoming too big and bulky.

The white operation displayed on each key is obtained by simply putting in the numbers and pressing the required key.

The orange operation, usually displayed in orange just above the key, is obtained by first pressing SHIFT (or INV), and then the required key.

In addition to giving a brief description of each of the keys, this chapter covers important information about the ORDER OF OPERATIONS and ROUNDING OFF.

ORDER OF OPERATIONS

Mathematicians around the world have agreed on a definite order of doing the four operations (+ − × ÷), otherwise confusion would occur.

For example, which is the true answer?

$6 + 3 \times 4 = 18$ OR $6 + 3 \times 4 = 36$

The correct answer is 18 because the multiplication must be done before the addition.

THE AGREED ORDER OF OPERATIONS:

> First, work out any grouping symbols or brackets.
> Second, work out any multiplication and division as they occur from left to right.
> Third, work out any addition and subtraction as they occur from left to right.

This is often remembered by students as:

B	**O**	**D**	**M**	**A**	**S**
Brackets	Of	Division	Multiplication	Addition	Subtraction

Most calculators should automatically follow the rules of BODMAS when the operation keys are pressed in the same order as the question – but this should be carefully checked with your calculator.

To check if your calculator has the order of operations built into it, do the calculation $14 - 6 \times 2 + 7$

i.e. [14] [−] [6] [×] [2] [+] [7] [=] 9

If you didn't get the answer of 9, then your calculator doesn't have BODMAS built into it, and you will then have to carry out the rules explained above.

FUNCTIONS OF THE DIFFERENT KEYS

CLEAR KEY: [C] or [AC] or [CA] (depending on model)

This key clears the entire problem from the calculator, except for any possible number stored in memory.

CLEAR ENTRY KEY: [DEL] or [CE] or [C] (depending on model)

This clears only the last number entered, and therefore it is a useful key for correcting input error without having to redo the whole problem.

Example: 65×132

Solution: [65] [×] [123] [CE] [132] [=] 8580

wrong number entered

ERROR: The calculator will lock and display the error symbol E (or MATH ERROR) under 2 conditions:

(i) An impossible calculation, such as $9 \div 0$ or $\sqrt{-16}$ is attempted. Cannot divide by zero in mathematics! Cannot calculate the square root of a negative number!

(ii) The answer lies outside the range of the calculator. This doesn't often occur as the range is extremely large usually going from 10^{-99} to 10^{99}.

NOTE: The calculator can be unlocked by pressing the CLEAR KEY.

SIGN CHANGE KEY: [±] Useful when doing problems which involve negative numbers. It changes the sign from positive to negative, or from negative to positive.

Example: $(-6.4) \div (-5.29)$

[6.4] [±] [÷] [5.29] [±] [=] 1.2098299

SQUARE ROOT: [$\sqrt{\ }$] Find the square root of a number.

Example: Find $\sqrt{1849}$

[$\sqrt{\ }$] [1849] [=] 43

SQUARE: [x^2] This key squares the number on the display screen.

Example: Find $(3.9)^2$

[3.9] [x^2] [=] 15.21

CUBE ROOT: [$\sqrt[3]{\ }$] Used to find the cube root of a number.

Example: Find $\sqrt[3]{343}$

[$\sqrt[3]{\ }$] [343] [=] 7

CUBE: [x^3] This key cubes the number on the display screen.

Example: Find $(3.9)^3$

[3.9] [x^3] [=] 59.319

POWER KEY: [x^y] or [$\wedge$] This key is used for powers higher than 3.

Example: Find 11^5

[11] [x^y] [5] [=] 161 051

ROOT KEY: $\sqrt[x]{\ }$ Used for finding roots higher than 2 or 3.

Example: Find $\sqrt[5]{7776}$

$\sqrt[x]{\ }$ 7776 = 6

THE RECIPROCAL KEY: $^1/_x$ or x^{-1} Used to invert a fraction, or turn it upside down. In many questions it will calculate $\frac{1}{\text{number}}$.

Example (i): Calculate $\frac{1}{0.8}$ 0.8 $^1/_x$ 1.25

The above problem could also have been done by division:

1 ÷ 0.8 = 1.25

Example (ii): $\frac{1}{4.6} - \frac{1}{7.3}$

4.6 x^{-1} – 7.3 x^{-1} = 0.080

FRACTION KEY: $\square\frac{\square}{\square}$ An important key used in combination with the direction keys to enter and perform operations with fractions and mixed numerals.

Example: Enter $5\frac{3}{4}$

$\square\frac{\square}{\square}$ 5 → 3 ↓ 4

BRACKETS: [(---] and [---)]

These keys are entered as in the normal arithmetic order. The left hand key is used to open the brackets, while the right hand key is used to close the brackets. A thorough understanding of when and how to use these keys is essential to ensure the correct order of operations.

Example (i): Evaluate $5.6 + 3.5 (6.4 – 2.4)$

[5.6] [+] [3.5] [(---] [6.4] [–] [2.4] [---)] [=] 19.6

Example (ii): Calculate $\frac{6}{0.3+0.2}$

[6] [÷] [(---] [0.3] [+] [0.2] [---)] [=] 12

NOTE: In this case, it is necessary to include the brackets to indicate that the 0.2 is part of the denominator of the fraction. Without the brackets the calculator would add the 0.2 after dividing the 6 by 0.3 and get 20.2

Example (iii): $\frac{12.473-6.849}{(1.46+2.84)^3}$

[(---] 12.473 [–] 6.849 [---)] [÷] [(---] 1.46 [+] 2.84 [---)] [x^3] [=]

0.0707359

The fraction key could also be used for this example, and will give the same answer when used correctly.

Example (iv): $\sqrt[3]{\frac{\pi \times (6.342)^2}{23.51-4.8}}$

[$\sqrt[3]{}$] [(---] [(---] [π] [×] 6.342 [x^2] [---)] [÷]

[(---] 23.51 [–] 4.8 [---)] [---)] [=] 1.8902077

ROUNDING OFF USING DECIMAL PLACES

Because of the everyday emphasis of the General course, there are many times when using a calculator that we need only an approximate answer. The question will usually state the accuracy required by telling you to round off to a specified number of decimal places.

All problems relating to money should be given rounded to 2 decimal places (or the nearest hundredth), because there are 100 cents in a dollar and the cents cannot be subdivided into smaller units. On some occasions you will be required to round money amounts to the nearest 5 cents.

To round off a number correct to a given decimal place:

- cut the number at the required decimal place;
- look at the digit immediately to the right of the specified place;
- if this digit is 0, 1, 2, 3 or 4, leave the number in the specified place unchanged;
- if the digit is 5, 6, 7, 8 or 9, add 1 to the number in the specified place.

Example (i): Round 7.264 correct to one decimal place.

7.2 | 64 (cut)

The next digit is 6, so add 1 to the 2 in the tenths place, to give 3.

So 7.264 is 7.3 (correct to 1 d.p).

NOTE: Another way of asking the same question is: "Round off to the nearest tenth".

Example (ii): Round 5.9327 correct to two decimal places.

5.93 | 27 (cut)

The next digit is 2, so the number 3 does not change.

So 5.9327 is 5.93 (correct to 2 d.p).

NOTE: In later model scientific calculators, the answer can be rounded off to a specified number of decimal places using the FIX function. When using the FIX function it is important to know how to reset your calculator back to NORMAL. Both these functions are usually found by pressing the MODE key.

SCIENTIFIC NOTATION – LARGE NUMBERS

When a number is too large to be shown on the display screen a calculator will convert the number to **scientific notation** (or standard notation)**.** Scientific notation is when a number is written as a number between 1 and 10, multiplied by a power of 10. This is a useful and quick way of writing very large or very small numbers.

Example (i): Write 6.37×10^5 as an ordinary numeral.

Solution: 6.37×10^5 means $6.37 \times 10\,000 = 6\,3\,7\,0\,0\,0$

The power of the 10 indicates the number of places to move the decimal point to the right.
$\therefore$ The decimal point has been moved 5 places to the right.

Example (ii): Write the answer for $8.269 \times 10\,000$ in scientific notation.

Solution: $8.269 \times 10\,000 = 8.269 \times 10^4$
When the number is written as a number between 1 and 10 multiplied by a multiple of 10, the number of zeros in the multiple of 10 is the same as the power of 10.

Example (iii): Write 9 320 000 in scientific notation.

Solution: $9\,320\,000 = 9.32 \times 1\,000\,000 = 9.32 \times 10^6$

The decimal point must be moved 6 places to the left so that the number is between 1 and 10.
The number 10 must therefore be raised to the power of 6 to compensate for the movement of the decimal point.

Example (iv): Express 6 700 in standard notation.

Solution: $6\,700 = 6.7 \times 10^3$ The decimal point has been moved 3 places to the left, therefore the 10 must be raised to the power of 3.

When using a scientific calculator you can enter numbers in scientific notation by using the [EXP] key.

Example (v): Enter 4.75×10^7

Solution: 4.75 [EXP] 7 Older calculators display this as [4.75 07] with a gap between the number and the power of 10.
Newer models can display [4.75×10^7]

REVIEW EXERCISE – LEVEL 1

1. Round off to two decimal places:

   (a) 12.374 (b) 0.86439 (c) 136.998 (d) 0.085

2. Round off to two significant figures:

   (a) 0.786 (b) 4.383 (c) 0.00783 (d) 453

3. Write these numbers in scientific notation:

   (a) 43 700 (b) 6 000 (c) 0.00075 (d) 0.1

4. Write these numbers in ordinary decimal form:

   (a) 5.68×10^{6} (b) 9×10^{-5} (c) 4.2×10^{-2} (d) 8×10^{3}

5. Calculate to two decimal places:

   (a) $5.6 + 4 \times 3.66$ (b) $\dfrac{6.9+3.7}{8.2}$

   (c) $24.9^2 + 7.62^2$ (d) $\dfrac{85.61-36.4}{42.3+18.75}$

   (e) $16.8^3 - 18.3^4$ (f) $\sqrt{3.5+2.8}$

   (g) $\sqrt{35^2+28^2}$ (h) $46\times38+\dfrac{1}{2}\times9.8\times7^2$

   (i) $(4.3)^{\frac{1}{5}}$ (j) $\sqrt[3]{8.2-10.4}$

6. Evaluate, leaving answers in scientific notation, correct to 3 significant figures:

   (a) $\left(8.7\times10^{9}\right)\times\left(4.3\times10^{2}\right)$ (b) $\dfrac{4.8\times10^{5}}{3.2\times10^{-7}}$

   (c) $\left(2\times10^{-4}\right)^{3}$ (d) $\left(5.15\times10^{5}\right)\times\left(2\times10^{-2}\right)$

   (e) $\dfrac{\left(3.8\times10^{-4}\right)^{2}\times\left(5\times10^{3}\right)}{9.5\times10^{-5}}$ (f) $\sqrt{\dfrac{4.2\times10^{3}}{9.7\times10^{-4}}}$

REVIEW EXERCISE – LEVEL 2

1. Round off in the way indicated:

   (a) 534 698 (to 3 significant figures)

   (b) 0.003 546 (to 2 decimal places)

   (c) 5.84×10^{-6} (to 2 significant figures)

   (d) 3.95×10^{-1} (to 2 decimal places)

2. Calculate, leaving answers in scientific notation to 2 significant figures:

   (a) $\pi \times (3.2 \times 10^{-2})^2$

   (b) $\sqrt{\left(4.1\times10^{3}\right)^{2}-\left(3\times10^{3}\right)^{2}}$

   (c) $(2.3 \times 10^{3}) + (9.8 \times 10^{-2}) + (4.5 \times 10^{3})$

3. Use the fraction key on your calculator to calculate:

   (a) $\frac{3}{4}+\frac{2}{3}-\frac{1}{2}$ (b) $5\frac{3}{5}\times4\frac{1}{7}$

   (c) $8\frac{1}{2}+3\frac{1}{4}\times\frac{2}{13}$ (d) $\frac{6\frac{1}{3}+5\frac{2}{5}}{8\frac{1}{4}-4\frac{2}{3}}$

   (e) $\frac{5}{6}$ of 624 (f) $\frac{\frac{7}{3}-\frac{2}{5}}{\frac{4}{7}\times\frac{3}{8}}$

4. Simplify the following, writing answers in standard notation:

   (a) $\frac{6.348\times10^{5}}{2.3\times10^{-3}\times3.1\times10^{4}}$ (b) $\frac{1.9\times10^{-3}\times2.4\times10^{4}}{8\times10^{4}}$

   (c) $\frac{3.98\times10^{4}\times\ 6.42\times10^{-5}}{1.592\times10^{-3}\times\ 1.07\times10^{7}}$ (d) $\frac{9.81\times10^{-3}\times\ 5.74\times10^{-6}}{2.87\times10^{2}\times\ 1.635\times10^{-4}}$

SOLUTIONS TO REVIEW EXERCISES

Chapter 1 The Calculator – Level 1

1. (a) 12.37 (b) 0.86 (c) 137.00 (d) 0.09
2. (a) 0.79 (b) 4.4 (c) 0.0078 (d) 450
3. (a) 4.37×10^4 (b) 6×10^3 (c) 7.5×10^{-4} (d) 1×10^{-1}
4. (a) 5 680 000 (b) 0.000 09 (c) 0.042 (d) 8 000
5. (a) 20.24 (b) 1.29 (c) 678.07 (d) 0.81
   (e) – 107409.68 (f) 2.51 (g) 44.82 (h) 1988.10
   (i) 1.34 (j) – 1.30
6. (a) 3.74×10^{12} (b) 1.50×10^{12} (c) 8.00×10^{-12} (d) 1.03×10^4
   (e) 7.60×10^0 (f) 2.08×10^3

Chapter 1 The Calculator – Level 2

1. (a) 535 000 (b) 0.00 (c) 5.8×10^{-6} (d) 0.40
2. (a) $0.0032169 \approx 3.2 \times 10^{-3}$ (b) $2794.6377 \approx 2.8 \times 10^3$
   (c) $6800.098 \approx 6.8 \times 10^3$
3. (a) $\frac{11}{12}$ (b) $23\frac{1}{5}$ (c) 9 (d) $3\frac{59}{215}$
   (e) 520 (f) $9\frac{1}{45}$
4. (a) 8.903×10^3 (b) 5.7×10^{-4} (c) 1.5×10^{-4} (d) 5.63×10^{-8}

Chapter 2 Percentages using the Calculator – Level 1

1. (a) 0.35 (b) 2.6 (c) 0.08 (d) 0.146 (e) 0.014
2. (a) 25% (b) 150% (c) $66\frac{2}{3}\%$ (d) 45.2% (e) 5%
3. (a) $(100 + 15)\% \times 450 = 1.15 \times 450 = 517.5$
   (b) $(100 + 22)\% \times \$35 = 1.22 \times \$35 = \$42.70$
   (c) $(100 + 20)\% \times 600$ mL $= 1.20 \times 600$ mL $= 1320$ mL or 1.32 L
4. (a) $(100 - 42)\% \times 250 = 0.58 \times 250 = 145$
   (b) $(100 - 8)\% \times 800$ mL $= 0.92 \times 800 = 736$ mL
   (c) $(100 - 25)\% \times \$54.60 = 0.75 \times \$54.60 = \$40.95$