UNDERSTANDING YEAR 7 MATHS

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CONTENTS

NOTE: The Australian National Curriculum has been split into 3 major strands:

Ⓐ Number & Algebra Ⓑ Measurement & Geometry Ⓒ Statistics & Probability

In the Year 7 content descriptions, these 3 major strands have been further subdivided into the sub-strands shown above. Also, because Real Numbers contains such a large volume of important information, we have split it into 3 different sections of work.

WHY ARE YEARS 7 & 8 SO IMPORTANT?

The first 2 years of High School are obviously critical years in a child's education, and particularly so in the important subject area of Mathematics. It must be remembered that the syllabus is spiral in nature. This means that every year, a substantial amount of time will be spent revising and consolidating on the previous years work, before any new ideas are presented. This continual revision and building on previous work is intended to enhance greater understanding through repetition, while at the same time it will help students to memorise work they may have forgotten. This consolidation is particularly important in Year 7 where the students have come from a variety of different primary schools, in which some major topics may not have been taught with the same emphasis or method, or some ideas may not have been covered. Therefore the first few months of Year 7 will be spent revising the most important work from Grade 6 (i.e. Whole numbers, Decimals, Percentages, Fractions, etc.).

In addition to consolidating and extending on the ideas covered in Grade 6, some important new ideas will also be introduced in Year 7. These include more difficult Fractions, Negative Numbers, Geometric Reasoning and Algebra. For possibly the first time definite rules, formulae and methods should be starting to develop. Many of the problems given cannot usually be done mentally as in Grade 6, and often several skills such as reading, comprehension, lateral thinking and possibly the use of several logical steps will be required to arrive at the final solution. The writing and setting out of these steps is also becoming increasingly more important. These ideas of Number, Algebra, Measurement, Geometric Reasoning, Problem Solving, and Statistics form the essential foundations on which all later ideas in Mathematics are built.

The other important factor that students and parents should know is that the topics covered in Year 7 and Year 8 are part of a two year syllabus. This means that the Department of Education has written down a detailed syllabus which all schools should try to complete by the end of Year 8. Since it is a two year syllabus, students sitting tests and exams at the end of Year 8 will also be automatically tested on all the work covered in Year 7. The results of these exams will in a large way determine which level of Maths a student is capable of studying in Years 9 and 10. At the beginning of Year 9 they will be streamed into 3 separate courses, each of which use a different textbook and follow a different syllabus.

Therefore it can be seen that Year 8 exams are vital, because once a student has been assigned to a particular course in Year 9, it then becomes very difficult to move up into a higher level. This in turn will affect the level of Maths they are capable of doing in Years 11 and 12, and ultimately their career options. If students are confident, achieving high results and enjoying the subject in Year 7, then it is highly likely that they will continue with the same success throughout their Middle and Senior High School years.

You will see me on many of the pages… I will be trying to give you some reminders and advice.

THE NEW NATIONAL AUSTRALIAN CURRICULUM

Warwick Marlin acknowledges the dedicated work of the Australian Curriculum, Assessment and Reporting Authority (ACARA) and the many others who have contributed to the development of the Australian curriculum in response to the aims of the 2008 Melbourne Declaration on Educational Goals for Young Australians.

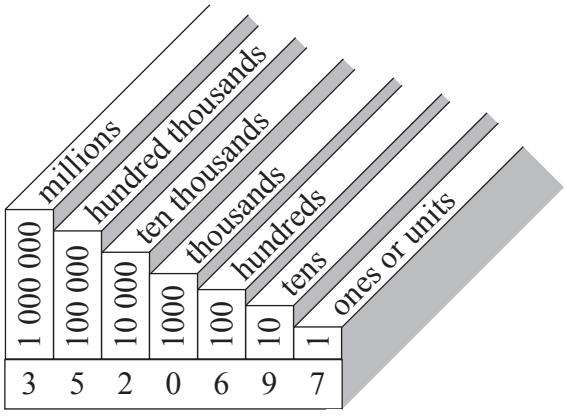
This book provides a summary and interpretation of their guidelines for those interested in developing mathematical understanding in Year 7 students.

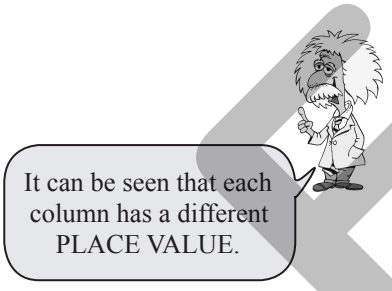
The Australian National Curriculum, developed by ACARA, states that, by the end of Year 7, students should be able to do the following:

- ❑ Solve problems involving the comparison, addition and subtraction of integers.
- ❑ Make the connections between whole numbers and index notation and the relationship between perfect squares and square roots.
- ❑ Solve problems involving percentages and all four operations with fractions and decimals.
- ❑ Compare the cost of items to make financial decisions.
- ❑ Represent numbers using variables.
- ❑ Connect the laws and properties for numbers to algebra.
- ❑ Interpret simple linear representations and model authentic information.
- ❑ Describe different views of three - dimensional objects.
- ❑ Represent transformations in the Cartesian plane.
- ❑ Solve simple numerical problems involving angles formed by a transversal crossing two parallel lines.
- ❑ Identify issues involving the collection of continuous data.
- ❑ Describe the relationship between the mean and median in data displays.
- ❑ Use fractions, decimals and percentages, and their equivalences.
- ❑ Express one quantity as a fraction or percentage of another.
- ❑ Solve simple linear equations.
- ❑ Evaluate algebraic expressions after numerical substitution.
- ❑ Assign ordered pairs to given points on the Cartesian plane.
- ❑ Use formulas for the area and perimeter of rectangles and calculate volumes of rectangular prisms.
- ❑ Classify triangles and quadrilaterals.
- ❑ Name the types of angles formed by a transversal crossing parallel lines.
- ❑ Determine the sample space for simple experiments with equally likely outcomes, and assign probabilities to those outcomes.
- ❑ Calculate mean, mode, median and range for data sets.
- ❑ Construct stem-and-leaf plots and dot plots.

PLACE VALUE

Since this chapter deals with WHOLE NUMBERS, the first important idea is to understand the value of each of the different columns. Our number system today is based on the Hindu-Arabic system where the VALUE of a number is determined by its PLACE in a particular column.
For example, what does 3 520 697 really mean?





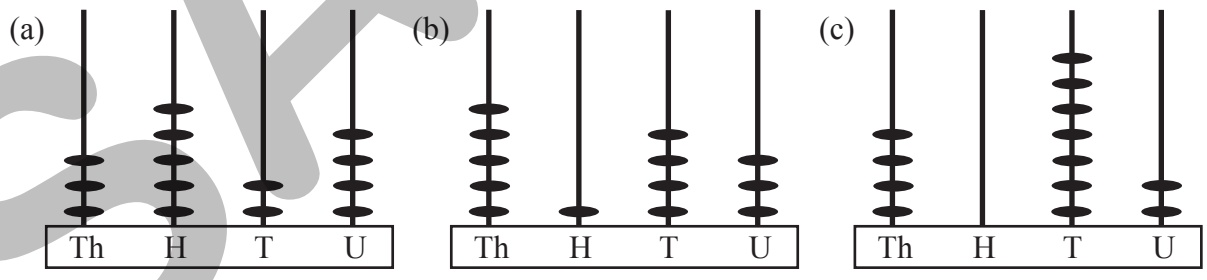
The place value of 9 is 90 or ninety.
The place value of 2 is 20 000 or twenty thousand.
The place value of 6 is 600 or six hundred.

There are 3 ways or notations of describing a whole number:

1. **AS AN ORDINARY NUMERAL:** 3 520 697
2. **IN WORDS:** Three million, five hundred and twenty thousand, six hundred and ninety seven.
3. **IN EXPANDED NOTATION:** $(3 \times 1\,000\,000) + (5 \times 100\,000) + (2 \times 10\,000) + (0 \times 1\,000) + (6 \times 100) + (9 \times 10) + (7 \times 1)$

You must be able to change from one notation to another.

Example:

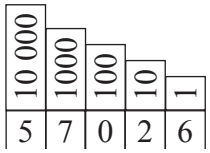


For each abacus above, write the numeral. Also write the numeral in words and in expanded notation.

Solutions:
(a) Numeral = 3 524 In words: Three thousand, five hundred & twenty four
In expanded notation: $(3 \times 1\,000) + (5 \times 100) + (2 \times 10) + (4 \times 1)$
(b) Numeral = 5 143 In words: Five thousand, one hundred & forty three
In expanded notation: $(5 \times 1\,000) + (1 \times 100) + (4 \times 10) + (3 \times 1)$
(c) Numeral = 4 072 In words: Four thousand and seventy two
In expanded notation: $(4 \times 1\,000) + (7 \times 10) + (2 \times 1)$

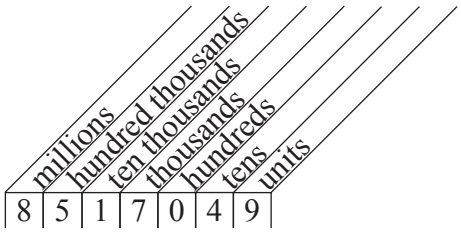
FURTHER EXAMPLES

Example 1: Write $(5 \times 10\,000) + (7 \times 1\,000) + (0 \times 100) + (2 \times 10) + (6 \times 1)$ as an ordinary numeral.



ANSWER: 57 026

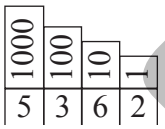
Example 2: Write eight million, five hundred and seventeen thousand and forty nine as an ordinary numeral.



ANSWER: 8 517 049

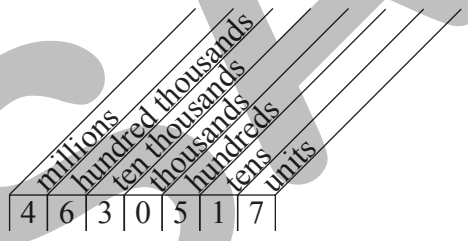
Example 3: Write 5 362 in expanded notation.

ANSWER: $(5 \times 1\,000) + (3 \times 100) + (6 \times 10) + (2 \times 1)$



You do not have to draw the diagrams to answer the questions. We have drawn them only to help you understand how the answers are obtained.

Example 4: Write 4 630 517 in words.



ANSWER: Four million, six hundred and thirty thousand, five hundred and seventeen.

Example 5: Find the value for each underlined digit in the numbers below:

(a) 3 <u>7</u>28 (b) <u>8</u> 635 (c) 2 <u>5</u>3 079 (d) 683 5<u>9</u>2

Solutions: (a) 700 (b) 8 000 (c) 50 000 (d) 90

FACTORS

A factor is a number that divides exactly into a counting number.

For example, 7 is a factor of 21 because it divides exactly into 21, with no remainder.

$$7\overline{)21}\quad 3$$

5 is NOT a factor of 13 because it doesn't divide exactly into 13.

$$5\overline{)13}\quad 2 \text{ r } 3$$

There is a remainder of 3 in this second example.

A factor is a number which leaves no remainder after division.

Example 1: Which of the numbers below is 3 a factor of?
18, 20, 24, 30, 37

Solution: $3\overline{)18}\quad 6$ $\quad 3\overline{)20}\quad 6 \text{ r } 2$ $\quad 3\overline{)24}\quad 8$ $\quad 3\overline{)30}\quad 10$ $\quad 3\overline{)37}\quad 12 \text{ r } 1$

3 is a factor of 18, 24, and 30.

Example 2: List all the factors of (a) 24 and (b) 36.

Solutions: (a) $24 = 24 \times 1$
$24 = 12 \times 2$
$24 = 8 \times 3$
$24 = 6 \times 4$

The factors of 24 are:
{1, 2, 3, 4, 6, 8, 12, 24}

Find all the pairs of numbers which multiply to give (a) 24 and (b) 36.

(b) $36 = 36 \times 1$
$36 = 18 \times 2$
$36 = 12 \times 3$
$36 = 9 \times 4$
$36 = 6 \times 6$

The factors of 36 are:
{1, 2, 3, 4, 6, 9, 12, 18, 36}

MULTIPLES

Multiples, as you would expect from your knowledge of multiplication tables, has something to do with multiplying.

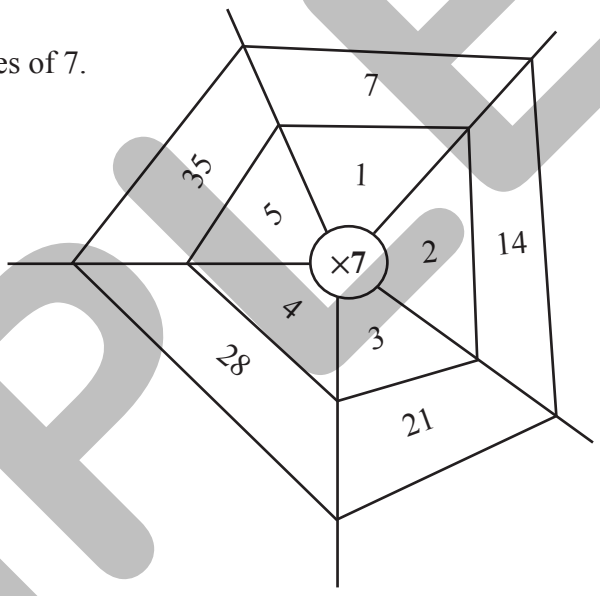
> To find the multiples of a particular number, simply multiply it by the counting numbers (1, 2, 3, 4, etc.).

Example 1: Say we wished to find the first 5 multiples of 7.

$1 \times 7 = 7$
$2 \times 7 = 14$
$3 \times 7 = 21$
$4 \times 7 = 28$
$5 \times 7 = 35$

The first 5 multiples of 7 are {7, 14, 21, 28, 35}.

OR
We can draw a WEB:



Example 2: List all the multiples of 5 which are less than or equal to 30.

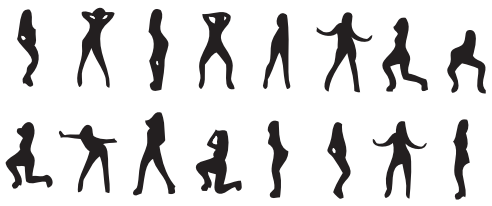
$1 \times 5 = 5$
$2 \times 5 = 10$
$3 \times 5 = 15$
$4 \times 5 = 20$
$5 \times 5 = 25$
$6 \times 5 = 30$

The multiples of 5 less than or equal to 30 are {5, 10, 15, 20, 25, 30}.

Example 3: List all the multiples of 6 which are between 20 and 45.

$1 \times 6 = 6$ $\quad 2 \times 6 = 12$ $\quad 3 \times 6 = 18$ $\quad 4 \times 6 = 24$
$5 \times 6 = 30$ $\quad 6 \times 6 = 36$ $\quad 7 \times 6 = 42$ $\quad 8 \times 6 = 48$

The multiples between 20 and 45 are {24, 30, 36, 42}



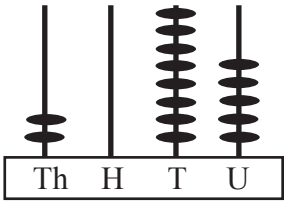
"Multiple silhouettes"

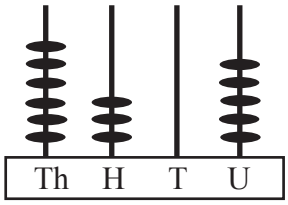
LEVEL 1 — NUMBER & PLACE VALUE

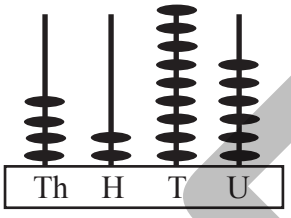
EASIER QUESTIONS

Note: Only turn back to page number shown if you have difficulty. Page

Q1. For each abacus below, write the numeral and also the numeral in words. 2

a) 

b) 

c) 

Q2. Write the following numbers in words: 2, 3

a) 732 b) 1 075 c) 32 756 d) 145 672

Q3. Write the following numbers as ordinary numerals: 2, 3

a) Seventy three thousand, five hundred & seven b) Fifteen thousand & six

Q4. Write the following as ordinary numerals: 2, 3

a) $(7\times1\,000)+(3\times100)+(5\times10)+(8\times1)$ b) $(9\times1\,000)+(6\times10)+(7\times1)$

Q5. Find the value for each underlined digit in the numbers below: 3

a) 6 $\underline{8}$39 b) $\underline{9}$ 271 c) 2$\underline{3}$7 481 d) $\underline{6}$8 752

Q6. List all the factors of the following numbers: 4

a) 9 b) 16 c) 24 d) 36

Q7. List the first five multiples of the following numbers: 5

a) 6 b) 7 c) 8 d) 9

Q8. Round off the following numbers to the nearest ten: 8

a) 37 b) 62 c) 75 d) 826

Q9. List the next 3 numbers in the following sequences: 9

a) 6, 12, 18, 24, ☐, ☐, ☐ b) 4, 9, 16, 25, ☐, ☐, ☐

c) 1, 3, 6, 10, ☐, ☐, ☐ d) 21, 19, 17, 15, ☐, ☐, ☐

Q10. Give the name for each set of numbers in Q9. 9

Q11. Write the following numbers using index notation: 14, 15

a) $7\times7\times7$ b) $6\times6\times6\times6$ c) 3×3 d) $10\times10\times10\times10\times10$

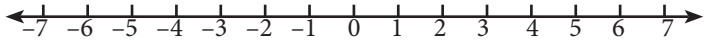
Q12. Write the following numbers as ordinary numerals: 14, 15

a) $(4\times10^3)+(5\times10^2)+(2\times10^1)+(7\times1)$ b) $(6\times10^4)+(8\times10^3)+(0\times10^2)+(9\times10^1)+(3\times1)$

Q13. Write the following numbers in index or exponential notation: 14, 15

a) 432 b) 6 509 c) 7 384 d) 15 462

Q14. By using a solid dot, graph the following numbers on the number line: 23, 24

{+5, –3, 0, –6, 2, –1} 

Q15. With the help of a number line, find: 23, 24

a) 3 – 7 = b) – 6 + 5 = c) – 4 + 7 = d) 5 – 7 =

e) 4 – 10 = f) – 3 + 8 = g) – 10 + 2 = h) – 6 – 2 =

LEVEL 1 – Number & Place Value

Q1. a) 2 085 Two thousand and eighty five
b) 6 305 Six thousand, three hundred and five
c) 4 296 Four thousand, two hundred and ninety six

Q2. a) Seven hundred and thirty two
b) One thousand and seventy five
c) Thirty two thousand, seven hundred & fifty six
d) One hundred & forty five thousand, six hundred & seventy two

Q3. a) 73 507
b) 15 006

Q4. a) 7 358
b) 9 067

Q5. a) 800
b) 9 000
c) 30 000
d) 60 000

Q6. a) {1, 3, 9}
b) {1, 2, 4, 8, 16}
c) {1, 2, 3, 4, 6, 8, 12, 24}
d) {1, 2, 3, 4, 6, 9, 12, 18, 36}

Q7. a) {6, 12, 18, 24, 30}
b) {7, 14, 21, 28, 35}
c) {8, 16, 24, 32, 40}
d) {9, 18, 27, 36, 45}

Q8. a) 40
b) 60
c) 80
d) 830

Q9. a) 30, 36, 42
b) 36, 49, 64
c) 15, 21, 28
d) 13, 11, 9

Q10. a) Multiples of 6
b) Square numbers
c) Triangular numbers
d) Odd numbers

Q11. a) 7^3
b) 6^4
c) 3^2
d) 10^5

Q12. a) 4 527
b) 68 093

Q13. a) $(4\times10^2) + (3\times10^1) + (2\times1)$
b) $(6\times10^3) + (5\times10^2) + (0\times10^1) + (9\times1)$
c) $(7\times10^3) + (3\times10^2) + (8\times10^1) + (4\times1)$
d) $(1\times10^4) + (5\times10^3) + (4\times10^2) + (6\times10^1) + (2\times1)$

Q14.

Q15. a) –4
b) –1
c) 3
d) –2
e) –6
f) 5
g) –8
h) –8

LEVEL 2 – Number & Place Value

Q1. a) 70
b) 6 000
c) 20 000
d) 700 000

Q2. a) {1, 2, 3, 6}
b) {1, 2, 4, 8}
c) {1, 2, 3, 4, 6, 12}
d) {1, 2, 3, 5, 6, 10, 15, 30}

Q3. a) 2
b) 4
c) 6
d) 6

Q4. a) {10, 20, 30, 40, 50}
b) {12, 24, 36, 48, 60, 72}
c) {25, 50, 75, 100, 125, 150, 175}
d) {15, 30, 45, 60, 75}

Q5. a) 20
b) 40
c) 12
d) 30

Q6. a) 10^3
b) 8^5
c) 4^6

Q7. a) 5 738
b) 60 593

Q8. a) $(6\times10^2) + (7\times10^1) + (3\times1)$
b) $(7\times10^3) + (8\times10^2) + (9\times10^1) + (2\times1)$
c) $(1\times10^4) + (2\times10^3) + (7\times10^2) + (3\times10^1) + (5\times1)$
d) $(7\times10^4) + (5\times10^3) + (0\times10^2) + (6\times10^1) + (2\times1)$

Q9. a) 3
b) 7
c) 10
d) 6

Q10. a) Commutative Law
b) Distributive Law
c) Associative Law

Q11. a) 0°C
b) –2°C
c) –6°C
d) –20°C
e) 0°C
f) 3°C
g) 3°C
h) –2°C

Q12. a) 0
b) –3
c) –4
d) –3
e) 4
f) 1
g) –5
h) –8

LEVEL 3 – Number & Place Value

Q1. a) 800
b) 80
c) 8 000
d) 800 000

Q2. a) 75 853 Seventy five thousand, eight hundred and fifty three
b) 6 830 678 Six million, eight hundred & thirty thousand, six hundred & seventy eight

Q3. a) 5
b) 6
c) 9
d) 15

Q4. a) 36
b) 24
c) 36
d) 60

Q5. a) 600
b) 800
c) 800
d) 3 500