UNDERSTANDING YEAR 8 MATHS

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CONTENTS

NOTE: The Australian National Curriculum has been split into 3 major strands:

Ⓐ Number & Algebra Ⓑ Measurement & Geometry Ⓒ Statistics & Probability

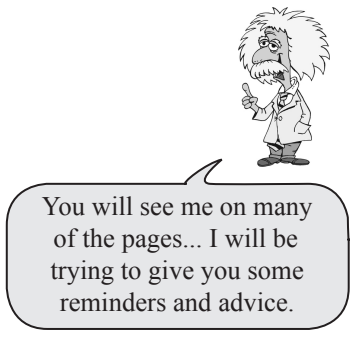
In the Year 8 content descriptions, these 3 major strands have been further subdivided into the sub-strands shown above.

WHY ARE YEARS 7 & 8 SO IMPORTANT?

In addition to consolidating and extending on the ideas covered in Year 7, many important new concepts will also be introduced in Year 8. Most of the problems given cannot be done mentally, and often several skills such as reading, comprehension, lateral thinking and the use of several logical steps will be required to arrive at the final solution. There is also a significant jump in the overall complexity of work from Year 7 to Year 8. Some problems can only be solved with the use of algebraic techniques.

The other important factor that students and parents should know is that the topics covered in Year 7 and Year 8 are part of a two year syllabus. This means that the Department of Education has written down a detailed syllabus which all schools should try to complete by the end of Year 8. Since it is a two year syllabus, students sitting tests and exams at the end of Year 8 will also be automatically tested on all the work covered in Year 7. The results of these exams will in a large way determine which level of Maths a student is capable of studying in Years 9 and 10. At the beginning of Year 9 they will be streamed into 3 separate courses, each of which use a different textbook and follow a different syllabus. The above factors have been taken into consideration in the development of this book. We start each chapter by firstly revising all the important ideas that should have been covered and understood in Year 7.

Therefore it can be seen that Year 8 exams are vital, because once a student has been assigned to a particular course in Year 9, it then becomes very difficult to move up into a higher level. This in turn will affect the level of Maths they are capable of doing in Years 11 and 12, and ultimately their career options. If students are confident, achieving high results and enjoying the subject in Year 8, then it is highly likely that they will continue with the same success throughout their Middle and Senior High School years.



"The best teacher is not the one who knows the most, but the one who is most capable of reducing knowledge to that simple compound of the obvious and wonderful..."

H.L. Menken

THE NEW NATIONAL AUSTRALIAN CURRICULUM

Warwick Marlin acknowledges the dedicated work of the Australian Curriculum, Assessment and Reporting Authority (ACARA) and the many others who have contributed to the development of the Australian curriculum in response to the aims of the 2008 Melbourne Declaration on Educational Goals for Young Australians.

This book provides a summary and interpretation of their guidelines for those interested in developing mathematical understanding in Year 8 students.

The Australian National Curriculum, developed by ACARA, states that, by the end of Year 8, students should be able to do the following:

- ❑ Solve everyday problems involving rates, ratios and percentages.
- ❑ Recognise index laws and apply them to whole numbers.
- ❑ Describe rational and irrational numbers.
- ❑ Solve problems involving profit and loss.
- ❑ Make connections between expanding and factorising algebraic expressions.
- ❑ Solve problems relating to the volumes of prisms.
- ❑ Make sense of time duration in real applications.
- ❑ Identify conditions for the congruence of triangles and deduce the properties of quadrilaterals.
- ❑ Model authentic situations with two-way tables and Venn diagrams.
- ❑ Choose appropriate language to describe events and experiments.
- ❑ Explain issues related to the collection of data and the effect of outliers on means and medians in that data.
- ❑ Use efficient mental and written strategies to carry out the four operations with integers.
- ❑ Simplify a variety of algebraic expressions.
- ❑ Solve linear equations.
- ❑ Graph linear relations on the Cartesian plane.
- ❑ Convert between units of measurement for area and volume.
- ❑ Perform calculations to determine the perimeter and area of parallelograms, rhombuses and kites.
- ❑ Name the features of circles and calculate the area and circumference of circles.
- ❑ Determine complementary events and calculate the sum of probabilities.

'The advancement and perfection of Mathematics are intimately connected with the prosperity of the state...'

Napolean

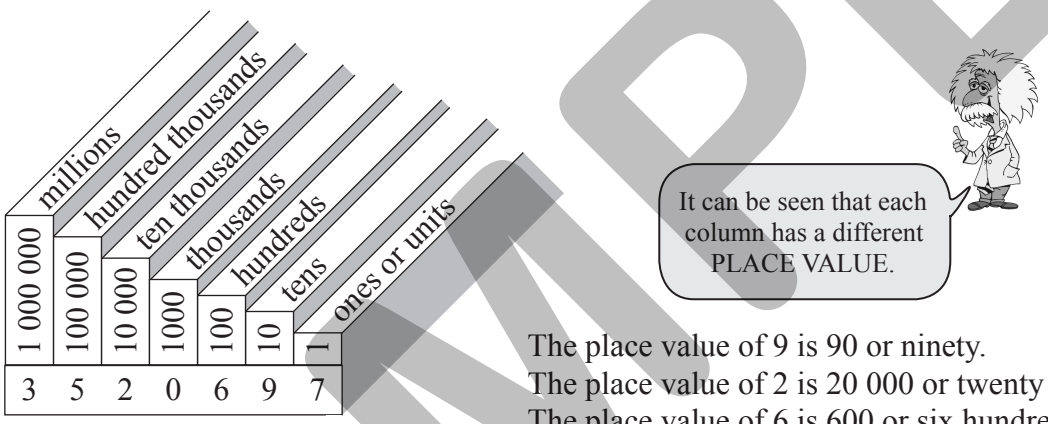
BRIEF SUMMARY OF YEAR 7

❑ INTRODUCTION

Please keep in mind that continual revision and reinforcement of important ideas from previous years is an essential part of the learning process. Therefore I have started this chapter with a very brief summary of some important words and ideas from Year 7. The exercises at the end of this chapter will primarily focus on the new Year 8 work as outlined by the ACARA statements and syllabus references at the top of the opening chapter page. However, there will also be some questions relating to the Year 7 work as outlined on the next few pages. If you require more comprehensive explanations, and more examples, then please refer to the 'Understanding Year 7' book by the same author.

❑ PLACE VALUE

Since this chapter is entitled 'NUMBER & PLACE VALUE', the first important idea is to understand the value of each of the different columns. Our number system today is based on the Hindu-Arabic system where the VALUE of a number is determined by its PLACE in a particular column. For example, what does 3 520 697 really mean?



The place value of 9 is 90 or ninety.
The place value of 2 is 20 000 or twenty thousand.
The place value of 6 is 600 or six hundred.

There are 3 ways or notations of describing a whole number:

1. AS AN ORDINARY NUMERAL: 3 520 697
2. IN WORDS: Three million, five hundred and twenty thousand, six hundred and ninety seven.
3. IN EXPANDED NOTATION: $(3 \times 1\,000\,000) + (5 \times 100\,000) + (2 \times 10\,000) + (0 \times 1\,000) + (6 \times 100) + (9 \times 10) + (7 \times 1)$

❑ FACTORS

A factor is a number which leaves no remainder after division.

The factors of 12 are {1, 2, 3, 4, 6, 12}.

❑ HIGHEST COMMON FACTOR (HCF)

This is the highest factor which is common to 2 or more numbers.

Example: Find the highest common factor of 12 and 20.
Factors of 12 are {1, 2, 3, 4, 6, 12}.
Factors of 20 are {1, 2, 4, 5, 10, 20}.
$\therefore$ The HCF of 12 and 20 is 4.

TERMINATING DECIMALS

We have thoroughly explained and covered all the important ideas relating to Fractions, Decimals and Percentages in Grade 6 and also again in Year 7. Therefore we shall begin with only a very brief revision of the previous work, and then start immediately on some new ideas.

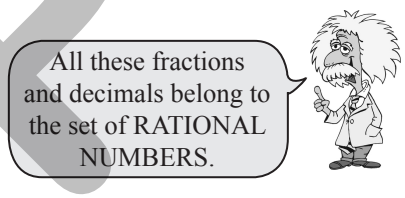
We shall start here with the very basic connection between simple fractions and decimals. The method shown below is used when it is fairly easy to find an equivalent fraction with a denominator which has a power of 10. The decimals shown on these 2 pages are all called TERMINATING DECIMALS because they all have a definite end point.

> (i) Change the fraction to an equivalent fraction which has a denominator of 10 or 100 or 1 000.
> (ii) Then simply convert the fraction to a decimal.

Example 1: Change $\frac{2}{5}$ to a decimal.

Convert $\frac{2}{5}$ to an equivalent fraction with a denominator of 10.

Therefore $\frac{2}{5} \times \frac{2}{2} = \frac{4}{10} = 0.4$



Example 2: Change $\frac{7}{20}$ to a decimal.

Convert $\frac{7}{20}$ to an equivalent fraction with a denominator of 100.

Therefore $\frac{7}{20} \times \frac{5}{5} = \frac{35}{100} = 0.35$

Example 3: Change $\frac{3}{4}$ to a decimal.

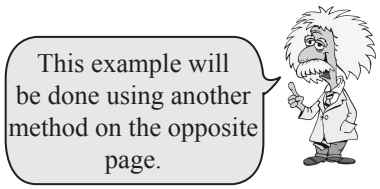
Convert $\frac{3}{4}$ to an equivalent fraction with a denominator of 100.

Therefore $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 0.75$

Example 4: Change $\frac{3}{8}$ to a decimal.

Convert $\frac{3}{8}$ to an equivalent fraction with a denominator of 1 000.

Therefore $\frac{3}{8} \times \frac{125}{125} = \frac{375}{1\,000} = 0.375$



CHANGING HARDER FRACTIONS TO DECIMALS

It is not always easy to make an equivalent fraction which has a denominator of 10 or 100 or 1 000 as was shown in the last example on the previous page. The method below will work for all conversions of fractions to decimals.

> (i) Write the numerator as a decimal (by adding a decimal point and zeros).
> (ii) Divide the decimal by the denominator.

Example 1: Change $\frac{3}{8}$ to a decimal.

Write 3 as a decimal = 3.000

$\frac{3}{8}$ is the same as 3.000 ÷ 8

$$8\overline{)3.^{3}0\,^{6}0\,^{4}0} \quad 0.\ 3\ 7\ 5$$

Therefore $\frac{3}{8} = 0.375$

Example 2: Change $\frac{13}{40}$ to a decimal.

Write 13 as a decimal = 13.000

$\frac{13}{40}$ is the same as 13.000 ÷ 40

$$40\overline{)13.^{13}0\,^{10}0\,^{20}0} \quad 0.\ 3\ 2\ 5$$

Therefore $\frac{13}{40} = 0.325$

Note: Check the above answers with the aid of your calculator.

13 [÷] 40 [=] 0.325

3 [÷] 8 [=] 0.375

REPEATING DECIMALS

When we do the division on the previous page, we sometimes get decimal answers that never end.

Example 1: Write $\frac{1}{3}$ as a decimal.

$$3\overline{)1.\,{}^{1}0\,{}^{1}0\,{}^{1}0\,{}^{1}0} \quad 0.\,3\,3\,3\,3\ldots.$$

$$\therefore \frac{1}{3} = 0.\dot{3}$$

We place a dot above the 3 to show that this number keeps on repeating infinitely.

> When a decimal keeps repeating, it is called a REPEATING or RECURRING DECIMAL.
> This is written with a dot on the repeating numbers.

Example 2: Change $\frac{5}{11}$ to a decimal.

Write 5 as a decimal = 5.000

$$11\overline{)5.{}^{6}0\,{}^{5}0\,{}^{6}0\,{}^{5}0\,{}^{6}0\,{}^{5}0} \quad 0.\,4\,5\,4\,5\,4\,5\ldots.$$

$$\Rightarrow \frac{5}{11} = 0.\dot{4}\dot{5}$$

We place a dot on both the 4 and 5 to show that the pattern 45 repeats itself infinitely.

Example 3: Change $\frac{254}{999}$ to a decimal.

Try doing this on your calculator and you should obtain the following answers:

254 ÷ 999 = 0.254254254254.....

To show that 254 is a repeating pattern we place a dot over the 2 and also over the 4.

$\therefore$ 0.254 254 254.... = $0.\dot{2}5\dot{4}$

LEVEL 1 — REAL NUMBERS

EASIER QUESTIONS

Note: Only turn back to page number shown if you have difficulty. Page

Q1. Change the following fractions to decimals: 34, 35

a) $\frac{7}{8}$ b) $\frac{17}{20}$ c) $\frac{29}{40}$ d) $\frac{53}{400}$

Q2. Change the following fractions to repeating decimals: 36

a) $\frac{7}{9}$ b) $\frac{3}{11}$ c) $\frac{5}{24}$ d) $\frac{7}{30}$

Q3. State whether the following numbers are rational or irrational: 37

a) $\frac{7}{25}$ b) 0.003917 c) $4.5\dot{7}$ d) $\sqrt{7}$

Q4. Change the following fractions to percentages: 38

a) $\frac{13}{20}$ b) $\frac{28}{400}$ c) $\frac{7}{8}$ d) $\frac{2}{3}$

Q5. What are the prices of the items below after calculating the given discounts: 41

a)  shoes $140 25 % discount

b)  wallet $29.50 10 % discount

c)  t-shirt $12.00 40 % discount

d)  belt $14.35 10 % discount

Q6. Increase the following amounts by the percentage shown: 44

a) increase $870.00 by 10%

b) increase $780.00 by 30%

c) increase 480 g by 4%

d) increase 3 400 m by 15%

Q7. Decrease the following amounts by the percentage shown: 45

a) decrease $320 by 25%

b) decrease $640.60 by 10%

c) decrease 700 km by 5%

d) decrease 800 sheep by 37%

Q8. A cake is made up of the following 5 ingredients: 180 g flour, 30 g sugar, 80 g chocolate, 50 g eggs and 60 g butter. Find the percentage composition of each ingredient. 43

Q9. Jenny gets $17 pocket money each week and her younger sister Kim gets $8 per week. Write down the ratio of: 47, 48

a) Jenny's pocket money to Kim's pocket money.
b) Kim's pocket money to Jenny's pocket money.
c) Kim's pocket money to the total pocket money given out.

Q10. Complete the following equivalent ratios: 47-49

a) 4 : 1 = 12 : ☐ b) 5 : 2 = 20 : ☐ c) 3 : 7 = 9 : ☐

d) 6 : 5 = ☐ : 15 e) 3 : 8 = ☐ : 40 f) 7 : 9 = ☐ : 36

LEVEL 1 – Real Numbers

Q1. a) 0.875 b) 0.85 c) 0.725 d) 0.1325

Q2. a) $0.\dot{7}$ b) $0.2\dot{7}$ c) $0.208\dot{3}$ d) $0.2\dot{3}$

Q3. a) rational b) rational c) rational d) irrational

Q4. a) 65% b) 7% c) $87\frac{1}{2}\%$ d) $66\frac{2}{3}\%$

Q5. a) \$105.00 b) \$26.55 c) \$7.20 d) \$12.92

Q6. a) \$957.00 b) \$1 014.00 c) 499.2 g d) 3 910 m

Q7. a) \$240 b) \$576.54 c) 665 m d) 504 sheep

Q8. a) flour = 45%, sugar = $7\frac{1}{2}\%$, chocolate = 20%, eggs = $12\frac{1}{2}\%$, butter = 15%

Q9. a) Jenny : Kim = 17 : 8 b) Kim : Jenny = 8 : 17 c) Kim : total = 8 : 25

Q10. a) 4 : 1 = 12 : 3 b) 5 : 2 = 20 : 8 c) 3 : 7 = 9 : 21
d) 6 : 5 = 18 : 15 e) 3 : 8 = 15 : 40 f) 7 : 9 = 28 : 36

LEVEL 2 – Real Numbers

Q1. a) rational b) irrational c) rational d) rational
e) rational f) rational g) rational h) irrational

Q2. a) \$74.80 b) \$54.45 c) \$71.17 d) \$47.14

Q3. a) If we are adding on 10% GST, this is exactly the same as increasing a quantity by a percentage, as explained on page 44.
For the tennis racket ⇒ \$68.00 × 110% = \$68.00 × 1.1 = \$74.80
For the fishing rod ⇒ \$49.50 × 110% = \$49.50 × 1.1 = \$54.45 etc.

Q4. a) \$700.00 b) \$2 650.00 c) copper = 60%, zinc = 40% d) 2 m

Q5. a) \$86.10

Q6. a) 4 : 5 b) 1 : 4 c) 3 : 4 d) 5 : 6 : 9
e) 3 : 4 f) m : 4 g) 2 : 3 h) 39 : 19

Q7. a) \$21.75/h b) 240 mL/sec c) 37 words/min d) \$4.32 /kg

Q8. a) girls : boys = 15 : 12 b) sheep : cows = 5 : 2 c) tutors : students = 2 : 10
d) flour : water = 1 : 3 e) cement : gravel : sand = 1 : 2 : 5

LEVEL 3 – Real Numbers

Q1. a) 0. 26 b) 0.04 c) 0.1625 d) 0.536

Q2. a) $0.7\dot{3}$ b) $0.\dot{8}5\dot{1}$ c) $0.\dot{9}5\dot{1}2\dot{1}$ d) $0.4\dot{7}$

Q3. a) irrational b) rational c) rational d) rational (=15)
e) irrational f) rational g) rational h) rational (=4)

Q4. a) $\frac{3}{8}$ b) $\frac{2}{25}$ c) $\frac{11}{25}$ d) $\frac{2}{3}$

Q5. Applying a discount to a price is exactly the same as decreasing an amount by a percentage as explained on page 45. If we subtract 25% from 100% we get 75%. Now all the shopkeeper has to do is find 75% or 0.75 of all his items.

Q6. i) 3 : 12 or 1 : 4 ii) 3 : 4 iii) 4 : 12 or 1 : 3 iv) 5 : 4
v) $\frac{12}{3}$ or $\frac{4}{1}$ vi) 3 : 4 : 5 vii) $\frac{4}{5}$ viii) 3 : 7 : 12

Q7. a) 5 : 9 b) 6 : 1 c) 1 : 4 d) 16 : 27

Q8. Liu receives \$1 621.20 and Chen receives \$1 158.00.

Q9. a) \$60/h b) \$1.50/kg c) 75 km/h d) 7 mL/sec

Q10. a) 75 km/h b) 120 km/h c) 96 km/h
d) 120 km/h e) 36 km/h