UNDERSTANDING YEAR 5 MATHS

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CONTENTS

NOTE: The Australian National Curriculum has been split into 3 major strands:

Ⓐ Number & Algebra Ⓑ Measurement & Geometry Ⓒ Statistics & Probability

In the Year 5 content descriptions, these 3 major strands have been further subdivided into the sub-strands shown above.

WHY ARE YEARS 5 AND 6 SO IMPORTANT?

These are obviously critical years in a child's education, and particularly so in the important subject area of Mathematics. For possibly the first time in their schooling, definite rules and methods are beginning to develop. It is in these years that students should be starting to become confident with the four basic operations (adding, subtracting, multiplying and dividing) as they apply to Whole Numbers, Decimals and Fractions. They will start learning about percentages and the important connection between fractions, decimals and percentages (e.g. ¼ = 0.25 = 25%). In addition, they should be starting to develop an understanding and feeling of different measurements within the metric system that we use in everyday living (length, mass, capacity, area, volume, etc.). A relatively new important major strand called Statistics & Probability will help them to understand and interpret statistical graphs. They will be given hundreds of mental type questions which only involve one step or process, however another skill which is becoming increasingly more important in Maths at this age is the ability to solve problems. These problems are usually too difficult to do mentally because they often involve several skills such as reading, comprehension, lateral thinking and possibly the use of several logical steps in order to arrive at the final answer.

These major ideas of Number & Algebra, Measurement & Geometry, Statistics & Probability form the foundations on which all later ideas are built. It is therefore important, if not vital, that students develop understanding, confidence and enjoyment in these formative years. If they achieve success in later primary school years, then it is highly likely that these skills will continue to grow and flourish in their senior schooling. In addition, there is a growing percentage of students who are trying to achieve exceptionally well in order to gain entrance, or possibly even scholarships, to selective and private schools.

Most parents are aware of these factors, and there are many thousands who would like to help, stimulate or extend their children towards greater enjoyment and success in the subject. However there are several reasons why they might not:

(i) They feel unqualified to help because it has been so long since they went to school.

(ii) Some parents were weak in Maths when they were at school and therefore feel that their children will automatically be weak as well.

(iii) Most, if not all parents, are capable of helping their children, but are unsure of what is important to know of the syllabus.

(iv) Parents can easily solve the majority of primary school problems, but are worried about teaching a method different to the one used by the teacher.

Outlined above are just some of the reasons why a well-structured book of this nature, which covers the syllabus topic by topic, is so urgently required. It has been especially researched and developed because of a need by students and parents, for a comprehensive, well-presented, easy to understand Maths summary book which covers the most important ideas in senior primary school Maths throughout Australia.

It has been called 'Understanding Year 5 All Levels' because it caters for the wide range of capabilities within different classes at different primary schools. Completing the first 3 or 4 graded exercises at the end of each chapter will ensure a very solid and good understanding for weak and average students. But in addition, I have added considerable extension theory and exercises to each chapter, in order to extend, challenge and stimulate the more gifted students.

THE NEW NATIONAL AUSTRALIAN CURRICULUM

Warwick Marlin acknowledges the dedicated work of the Australian Curriculum, Assessment and Reporting Authority (ACARA) and the many others who have contributed to the development of the Australian Curriculum in response to the aims of the 2008 Melbourne Declaration on Educational Goals for Young Australians.

This book provides a summary and interpretation of their guidelines for those interested in developing mathematical understanding in year 5 students.

The Australian National Curriculum, developed by ACARA, states that, by the end of Year 5, students should be able to do the following:

- ❑ Solve simple problems involving the four operations using a range of strategies.
- ❑ Check the reasonableness of answers using estimation and rounding.
- ❑ Identify and describe factors and multiples.
- ❑ Explain plans for simple budgets.
- ❑ Connect three-dimensional objects with their two-dimensional representations.
- ❑ Describe transformations of two-dimensional shapes and identify line and rotational symmetry.
- ❑ Compare and interpret different data sets.
- ❑ Order decimals and fractions and locate them on number lines.
- ❑ Add and subtract fractions with the same denominator.
- ❑ Continue patterns by adding and subtracting fractions and decimals.
- ❑ Find unknown quantities in number sentences.
- ❑ Use appropriate units of measurement for length, area, volume, capacity and mass, and calculate perimeter and area of rectangles.
- ❑ Convert between 12 and 24 hour time.
- ❑ Use a grid reference system to locate landmarks.
- ❑ Measure and construct different angles.
- ❑ List outcomes of chance experiments with equally likely outcomes and assign probabilities between 0 and 1.
- ❑ Pose questions to gather data, and construct displays appropriate for the data.
- ❑ Describe the enlargement transformation.
- ❑ Recognise the GST component on invoices.

You will see me on many of the pages…I will be trying to give you some reminders and advice.

ANCIENT NUMBER SYSTEMS

Many thousands of years ago, when people first started living in communities and tribes, it became increasingly necessary and important to have a counting system. For example, shepherds had to know how many sheep, goats or cattle they were looking after. The early trades had to be able to count beads, precious stones, silks, money and other items. Different civilisations (Mayas, Greeks, Chinese, Egyptians, Babylonians, Hindus, Arabs, etc.) developed their own counting systems over the centuries. Many of these counting systems were eventually discarded in favour of more advanced systems.

For example, the early **EGYPTIAN** system is shown below:

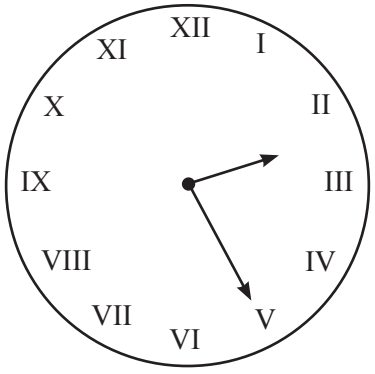
1	2	3	4	5	6	7	8	9	10	20	100
|	||	|||	||||	||| ||	||| |||	|||| |||	|||| ||||	||| ||| |||	∩	∩∩	𓆼

One of the most ancient systems was developed many thousands of years ago by the **MAYA** civilisation who lived in central America. It was quite a clever system based on 3 symbols:
a dot •, a straight line — and an oval with a v inside it ⓥ.

1	2	3	4	5	6	7	8	9	10	11	12	20	etc.
•	••	•••	⁘	—	• —	•• —	••• —	⁘ —	═	• ═	•• ═	ⓥ	

Our number system which is in use today (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc.) is called **HINDU-ARABIC** numerals. Archaeologists and historians believe it was first developed about 2300 years ago. (300 B.C.) in India. This counting system was then taken by trades and scholars to Arabia. The Arabs invaded Spain in the eighth century, and the Hindu-Arabic system was widely used throughout Europe by 1100 A.D.

One old system which is still widely used in this day and age is the **ROMAN** number system. Many clocks (particularly antiques and grandfather clocks) still have Roman numerals on their faces. Also you may have seen old buildings (such as churches, town halls and courthouses) which have the year they were built written on them in large Roman numerals.



ROMAN NUMERALS

When the Roman empire became powerful, its number system spread to many other countries. The system used a subtraction and addition idea. When a smaller unit appears before a larger one, it is subtracted from the larger one. When a smaller unit appears after the larger unit, it is added to the larger unit. Therefore the position of the symbols is important.

IV means 5 – 1 = 4 whereas VI means 5 + 1 = 6
XL means 50 – 10 = 40 whereas LX means 50 + 10 = 60

1	2	3	4	5	6	7	8	9
I	II	III	IV	V	VI	VII	VIII	IX
10	20	30	40	50	60	70	80	90
X	XX	XXX	XL	L	LX	LXX	LXXX	XC
100	200	300	400	500	600	700	800	900
C	CC	CCC	CD	D	DC	DCC	DCCC	CM

1 000 = M

You will often get questions in tests which ask you to change from Roman to Hindu-Arabic, and vice versa.

Example 1: Change the Roman numerals into our own numerals:

(a) XI (b) XXIV (c) LXXIII (d) DCLXIV

Solutions:

(a) X + I
= 10 + 1
= 11

(b) X + X + IV
= 10 + 10 + 4
= 24

(c) L + X +X + III
= 50 + 10 + 10 + 3
= 73

(d) D + C + L + X + IV
= 500 +100 + 50 + 10 + 4
= 664

Example 2: Change these Hindu-Arabic numerals into Roman numerals:

(a) 8 (b) 46 (c) 214 (d) 594

Solutions:

(a) 5 + 3
= V + III
= VIII

(b) 40 + 6
= XL + VI
= XLVI

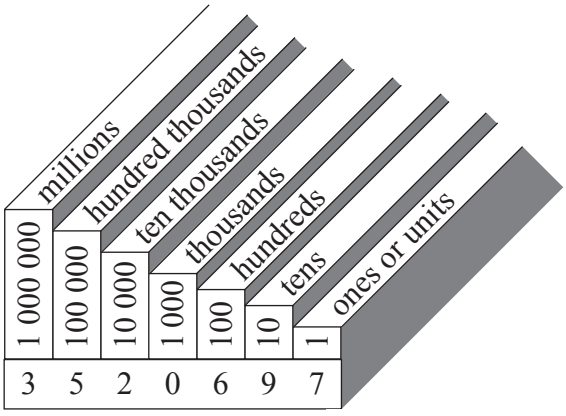
(c) 200 + 10 + 4
= CC + X + IV
= CCXIV

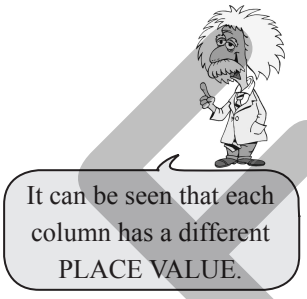
(d) 500 + 90 + 4
= D + XC + IV
= DXCIV

PLACE VALUE

Since this chapter deals with WHOLE NUMBERS, the first important idea is to understand the value of each of the different columns. Our number system today is based on the Hindu-Arabic system where the VALUE of a number is determined by its PLACE in a particular column.

For example, what does 3 520 697 really mean?





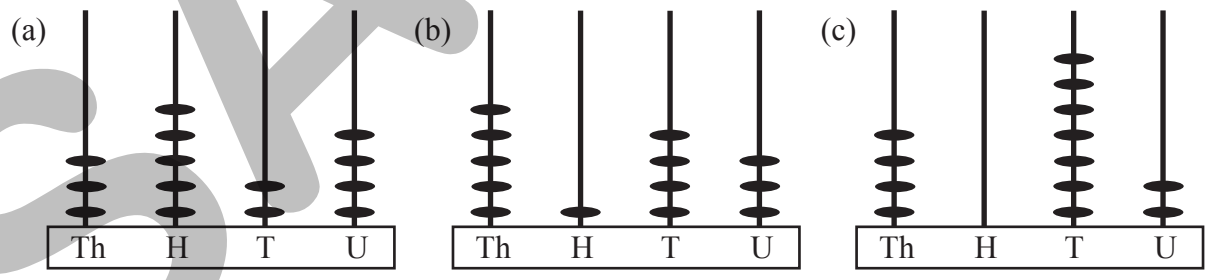
The place value of 9 is 90 or ninety.
The place value of 2 is 20 000 or twenty thousand.
The place value of 6 is 600 or six hundred.

There are 3 ways or notations of describing a whole number:

1. **AS AN ORDINARY NUMERAL:** 3 520 697
2. **IN WORDS:** Three million, five hundred and twenty thousand, six hundred and ninety-seven.
3. **IN EXPANDED NOTATION:** $(3 \times 1\,000\,000) + (5 \times 100\,000) + (2 \times 10\,000) + (0 \times 1\,000) + (6 \times 100) + (9 \times 10) + (7 \times 1)$

You must be able to change from one notation to another.

Example:



For each abacus above, write the numeral. Also write the numeral in words and in expanded notation.

Solutions: (a) Numeral = 3 524 In words: Three thousand five hundred and twenty-four
In expanded notation: $3 \times 1\,000 + 5 \times 100 + 2 \times 10 + 4$

(b) Numeral = 5 143 In words: Five thousand one hundred and forty-three
In expanded notation: $5 \times 1\,000 + 1 \times 100 + 4 \times 10 + 3$

(c) Numeral = 4 072 In words: Four thousand and seventy-two
In expanded notation: $4 \times 1\,000 + 7 \times 10 + 2$

FURTHER EXAMPLES

Example 1: Write $(5 \times 10\,000) + (7 \times 1\,000) + (0 \times 100) + (2 \times 10) + (6 \times 1)$ as an ordinary numeral.

10 000	1 000	100	10	1
5	7	0	2	6

ANSWER: 57 026

Example 2: Write eight million five hundred and seventeen thousand and forty-nine as an ordinary numeral.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	units
8	5	1	7	0	4	9

ANSWER: 8 517 049

Example 3: Write 5 362 in expanded notation.

1 000	100	10	1
5	3	6	2

ANSWER: (5 x 1 000) + (3 x 100) + (6 x 10) + (2 x 1)

You do not have to draw the diagrams to answer the questions. We have drawn them only to help you understand how the answers are obtained.

Example 4: Write 4 630 517 in words.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	units
4	6	3	0	5	1	7

ANSWER: Four million, six hundred and thirty thousand, five hundred and seventeen.

Example 5: Find the value for each underlined digit in the numbers below:

(a) 3 <u>7</u>28 (b) <u>8</u> 635 (c) 2<u>5</u>3 079 (d) 683 5<u>9</u>2

Solutions: (a) 700 (b) 8 000 (c) 50 000 (d) 90

LEVEL 2 — NUMBER AND PLACE VALUE

EASIER QUESTIONS

Note: Only turn back to page number shown if you have difficulty.

Page

Q1. Write the following as Hindu-Arabic numerals (our present number system): 2, 3

(a) VIII (b) XXI (c) IX (d) XXVI (e) LX

Q2. Write the following as Roman numerals: 2, 3

(a) 4 (b) 15 (c) 23 (d) 56 (e) 70

Q3. Write the following numerals in words: 4, 5

(a) 147 (b) 306 (c) 470 (d) 1 792

Q4. Write the following as ordinary numerals: 4, 5

(a) One hundred and fifty-three (b) Six hundred and twenty-eight
(c) Four hundred and nine (d) One thousand and forty

Q5. Write the following as ordinary numerals: 4, 5

(a) $(5 \times 100) + (7 \times 10) + (4 \times 1)$ (b) $(8 \times 100) + (0 \times 10) + (5 \times 1)$

Q6. Write the following numbers in expanded notation: 4, 5

(a) 452 (b) 748 (c) 590 (d) 803

Q7. List all the factors of the following numbers: 6

(a) 8 (b) 15 (c) 18 (d) 24 (e) 30

Q8. (a) List the first five multiples of 4. 7

(b) List the first four multiples of 6.

(c) List the multiples of 7 between 10 and 30.

Q9. (a) Is 58 closer to 50 or 60? Now round off 58 to the nearest 10. 10

(b) Is 94 closer to 90 or 100? Now round off 90 to the nearest 10.

(c) Is 35 closer to 30 or 40? Now round off 35 to the nearest 10.

Q10. Find: (a) $51 + 36$ (b) $47 + 15$ (c) $83 + 42$ (d) $67 + 85$ 14

Q11. Find: (a) $79 - 43$ (b) $85 - 42$ (c) $95 - 35$ (d) $74 - 25$ 15

Q12. Find: (a) 17×3 (b) 12×8 (c) 16×7 (d) 38×4 16

Q13. Find: (a) $5\overline{)45}$ (b) $3\overline{)72}$ (c) $4\overline{)60}$ (d) $7\overline{)105}$ 17

Q14. Find: 18

(a) sum of 13 and 8 (b) difference between 25 and 13 (c) product of 5 and 8

(d) average of 14 and 18 (e) quotient of 10 and 2 (f) 8 squared

LEVEL 1 – NUMBER AND PLACE VALUE

Q1. (a) 11 (b) 17 (c) 26 (d) 53 (e) 34

Q2. (a) VIII (b) XIII (c) XVIII (d) XIX (e) XLII

Q3. (a) One hundred and thirty two (b) Seven hundred and nine
(c) Five hundred and sixty (d) One thousand, five hundred and forty two

Q4. (a) 396 (b) 532

Q5. (a) 754 (b) 347

Q6. (a) {1, 2, 3, 6} (b) {1, 2, 5, 10} (c) {1, 2, 4, 5, 10, 20} (d) {1, 13}

Q7. (a) {5, 10, 15, 20} (b) {10,20,30,40, 50, 60} (c) {12, 15, 18, 21}

Q8. (a) 60 (b) 90 (c) 20

Q9. (a) 79 (b) 93 (c) 118 (d) 146

Q10. (a) 22 (b) 36 (c) 40 (d) 46

Q11. (a) 60 (b) 102 (c) 126 (d) 266

Q12. (a) 5 (b) 14 (c) 13 (d) 12

Q13. (a) 12 (b) 11 (c) 7 (d) 2 (e) 8 (f) 25

Q14. (a) 900 (b) 2 000 (c) 90 (d) 20 000

LEVEL 2 – NUMBER AND PLACE VALUE

Q1. (a) 8 (b) 21 (c) 9 (d) 26 (e) 60

Q2. (a) IV (b) XV (c) XXIII (d) LVI (e) LXX

Q3. (a) One hundred and forty seven (b) Three hundred and six
(c) Four hundred and seventy (d) One thousand, seven hundred and ninety two

Q4. (a) 153 (b) 628 (c) 409 (d) 1 040

Q5. (a) 574 (b) 805

Q6. (a) $(4 \times 100) + (5 \times 10) + (2 \times 1)$ (b) $(7 \times 100) + (4 \times 10) + (8 \times 1)$
(c) $(5 \times 100) + (9 \times 10) + (0 \times 1)$ (d) $(8 \times 100) + (0 \times 10) + (3 \times 1)$

Q7. (a) {1, 2, 4, 8} (b) {1, 3, 5, 15} (c) {1, 2, 3, 6, 9, 18}
(d) {1, 2, 3, 4, 6, 8, 12, 24} (e) {1, 2, 3, 5, 6, 10, 15, 30}

Q8. (a) {4, 8, 12, 16, 20} (b) {6, 12, 18, 24} (c) {14, 21, 28}

Q9. (a) 60 (b) 90 (c) 40 (the number 5 is rounded up !)

Q10. (a) 87 (b) 62 (c) 125 (d) 152

Q11. (a) 36 (b) 43 (c) 60 (d) 49

Q12. (a) 51 (b) 96 (c) 112 (d) 152

Q13. (a) 9 (b) 24 (c) 15 (d) 15

Q14. (a) 21 (b) 12 (c) 40 (d) 16 (e) 5 (f) 64