UNDERSTANDING YEAR 6 MATHS

SAMPLE

Author

Warwick Marlin B.Sc. Dip.Ed.

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NOTE: The Australian National Curriculum has been split into 3 major strands:

Ⓐ Number & Algebra Ⓑ Measurement & Geometry Ⓒ Statistics & Probability

In the Year 6 content descriptions, these 3 major strands have been further subdivided into the sub-strands shown above. Also, because 'Fractions and Decimals' contains such a large volume of important information, we have split it into 2 different sections of work.

Understanding Year 6 Maths
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WHY ARE YEARS 5 AND 6 SO IMPORTANT?

These are obviously critical years in a child's education, and particularly so in the important subject area of Mathematics. For possibly the first time in their schooling, definite rules and methods are beginning to develop. It is in these years that students should be starting to become confident with the four basic operations (adding, subtracting, multiplying and dividing) as they apply to Whole Numbers, Decimals and Fractions. They will start learning about percentages and the important connection between fractions, decimals and percentages (e.g. ¼ = 0.25 = 25%). In addition, they should be starting to develop an understanding and feeling of different measurements within the metric system that we use in everyday living (length, mass, capacity, area, volume, etc.). A relatively new important major strand called Statistics & Probability will help them to understand and interpret statistical graphs. They will be given hundreds of mental type questions which only involve one step or process, however another skill which is becoming increasingly more important in Maths at this age is the ability to solve problems. These problems are usually too difficult to do mentally because they often involve several skills such as reading, comprehension, lateral thinking and possibly the use of several logical steps in order to arrive at the final answer.

These major ideas of Number & Algebra, Measurement & Geometry, Statistics & Probability form the foundations on which all later ideas are built. It is therefore important, if not vital, that students develop understanding, confidence and enjoyment in these formative years. If they achieve success in later primary school years, then it is highly likely that these skills will continue to grow and flourish in their senior schooling. In addition, there is a growing percentage of students who are trying to achieve exceptionally well in order to gain entrance, or possibly even scholarships, to selective and private schools.

Most parents are aware of these factors, and there are many thousands who would like to help, stimulate or extend their children towards greater enjoyment and success in the subject. However there are several reasons why they might not:

(i) They feel unqualified to help because it has been so long since they went to school.

(ii) Some parents were weak in Maths when they were at school and therefore feel that their children will automatically be weak as well.

(iii) Most, if not all parents, are capable of helping their children, but are unsure of what is important to know of the syllabus.

(iv) Parents can easily solve the majority of primary school problems, but are worried about teaching a method different to the one used by the teacher.

Outlined above are just some of the reasons why a well-structured book of this nature, which covers the syllabus topic by topic, is so urgently required. It has been especially researched and developed because of a need by students and parents, for a comprehensive, well-presented, easy to understand Maths summary book which covers the most important ideas in senior primary school Maths throughout Australia.

It has been called 'Understanding Year 6 All Levels' because it caters for the wide range of capabilities within different classes at different primary schools. Completing the first 3 or 4 graded exercises at the end of each chapter will ensure a very solid and good understanding for weak and average students. But in addition, I have added considerable extension theory and exercises to each chapter, in order to extend, challenge and stimulate the more gifted students.

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THE NEW AUSTRALIAN CURRICULUM

Warwick Marlin acknowledges the dedicated work of the Australian Curriculum, Assessment and Reporting Authority (ACARA) and the many others who have contributed to the development of the Australian Curriculum in response to the aims of the 2008 Melbourne Declaration on Educational Goals for Young Australians.

This book provides a summary and interpretation of their guidelines for those interested in developing mathematical understanding in Year 6 students.

The Australian National Curriculum, developed by ACARA, states that, by the end of Year 6, students should be able to do the following:

- ❑ Recognise the properties of prime, composite, square and triangular numbers.
- ❑ Describe the use of integers in everyday contexts.
- ❑ Solve problems involving all four operations with whole numbers.
- ❑ Connect fractions, decimals and percentages as different representations of the same number.
- ❑ Solve problems involving addition and substraction of related fractions.
- ❑ Make connections between the powers of 10 and the multiplication and division of decimals.
- ❑ Describe rules used in sequences involving whole numbers, fractions and decimals.
- ❑ Connect decimal representations to the metric system and choose appropriate units of measurement to perform a calculation.
- ❑ Make connections between capacity and volume.
- ❑ Solve problems involving length and area.
- ❑ Interpret timetables.
- ❑ Describe combinations of transformations.
- ❑ Solve problems using the properties of angles.
- ❑ Compare observed and expected frequencies.
- ❑ Interpret and compare a variety of data displays including those displays for two categorical variables.
- ❑ Evaluate secondary data displayed in the media.
- ❑ Locate fractions and integers on a number line.
- ❑ Calculate a simple fraction of a quantity.
- ❑ Add, subtract and multiply decimals and divide decimals where the result is rational.
- ❑ Calculate common percentage discounts on sale items.
- ❑ Write correct number sentences using brackets and order of operations.
- ❑ Locate an ordered pair in any one of the four quadrants on the Cartesian plane.
- ❑ Construct simple prisms and pyramids.
- ❑ List and communicate probabilities using simple fractions, decimals and percentages.

You will see me on many of the pages... I will be trying to give you some reminders and advice.

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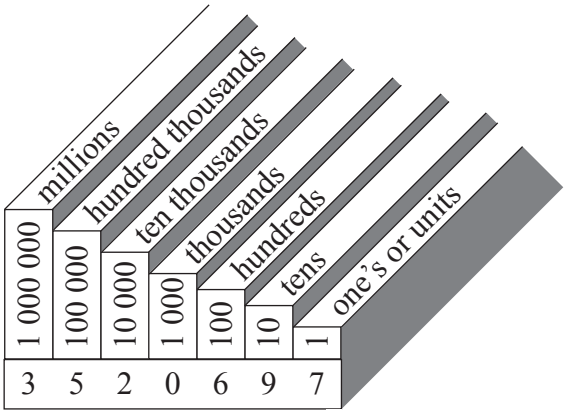
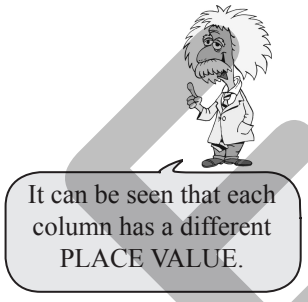
PLACE VALUE

Since this chapter deals with WHOLE NUMBERS, the first important idea is to understand the value of each of the different columns. Our number system today is based on the Hindu-Arabic system where the VALUE of a number is determined by its PLACE in a particular column.

For example, what does 3 520 697 really mean?





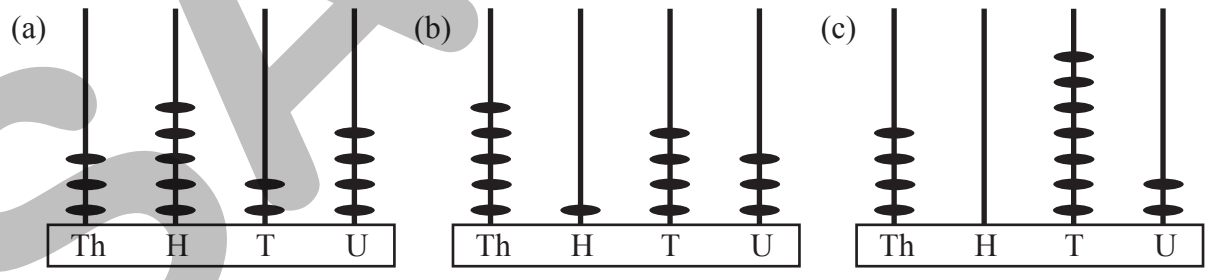
The place value of 9 is 90 or ninety.
The place value of 2 is 20 000 or twenty thousand.
The place value of 6 is 600 or six hundred.

There are 3 ways or notations of describing a whole number:

1. **AS AN ORDINARY NUMERAL:** 3 520 697
2. **IN WORDS:** Three million, five hundred and twenty thousand, six hundred and ninety seven.
3. **IN EXPANDED NOTATION:** $(3 \times 1\,000\,000) + (5 \times 100\,000) + (2 \times 10\,000) + (0 \times 1\,000) + (6 \times 100) + (9 \times 10) + (7 \times 1)$

You must be able to change from one notation to another.

Example:



For each abacus above, write the numeral. Also write the numeral in words and in expanded notation.

Solutions: (a) Numeral = 3 524 In words: Three thousand five hundred & twenty four
In expanded notation: $3 \times 1\,000 + 5 \times 100 + 2 \times 10 + 4$

(b) Numeral = 5 143 In words: Five thousand one hundred & forty three
In expanded notation: $5 \times 1\,000 + 1 \times 100 + 4 \times 10 + 3$

(c) Numeral = 4 072 In words: Four thousand and seventy two
In expanded notation: $4 \times 1\,000 + 7 \times 10 + 2$

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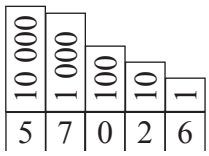
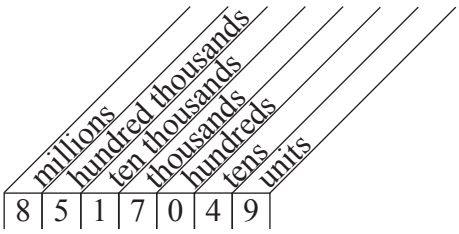
FURTHER EXAMPLES

Example 1: Write $(5 \times 10\,000) + (7 \times 1\,000) + (0 \times 100) + (2 \times 10) + (6 \times 1)$ as an ordinary numeral.



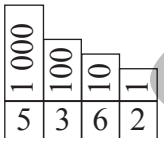
ANSWER: 57 026

Example 2: Write eight million five hundred and seventeen thousand and forty nine as an ordinary numeral.



ANSWER: 8 517 049

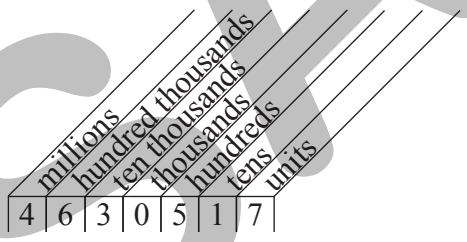
Example 3: Write 5 362 in expanded notation.



ANSWER: (5 x 1 000) + (3 x 100) + (6 x 10) + (2 x 1)

You do not have to draw the diagrams to answer the questions. We have drawn them only to help you understand how the answers are obtained.

Example 4: Write 4 630 517 in words.



ANSWER: Four million, six hundred and thirty thousand, five hundred and seventeen.

Example 5: Find the value for each underlined digit in the numbers below:

(a) 3 <u>7</u>28 (b) <u>8</u> 635 (c) 2<u>5</u>3 079 (d) 683 5<u>9</u>2

Solutions: (a) 700 (b) 8 000 (c) 50 000 (d) 90

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FACTORS

A factor is a number that divides exactly into a counting number.

For example, 7 is a factor of 21 because it divides exactly into 21, with no remainder.

$$7\overline{)21} = 3$$

5 is NOT a factor of 13 because it doesn't divide exactly into 13.

$$5\overline{)13} = 2 \text{ r } 3$$

There is a remainder of 3 in this second example.

A factor is a number which leaves no remainder after division.

Example 1: Which of the numbers below is 3 a factor of?
18, 20, 24, 30, 37

Solutions: $3\overline{)18} = 6$ $\quad 3\overline{)20} = 6 \text{ r } 2$ $\quad 3\overline{)24} = 8$ $\quad 3\overline{)30} = 10$ $\quad 3\overline{)37} = 12 \text{ r } 1$

3 is a factor of 18, 24, and 30.

Example 2: List all the factors of (a) 24 and (b) 36.

Solutions: (a) $24 = 24 \times 1$
$24 = 12 \times 2$
$24 = 8 \times 3$
$24 = 6 \times 4$

The factors of 24 are:
{1, 2, 3, 4, 6, 8, 12, 24}

Find all the pairs of numbers which multiply to give (a) 24 and (b) 36.

(b) $36 = 36 \times 1$
$36 = 18 \times 2$
$36 = 12 \times 3$
$36 = 9 \times 4$
$36 = 6 \times 6$

The factors of 36 are:
{1, 2, 3, 4, 6, 9, 12, 18, 36}

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MULTIPLES

Multiples, as you would expect from your knowledge of multiplication tables, has something to do with multiplying.

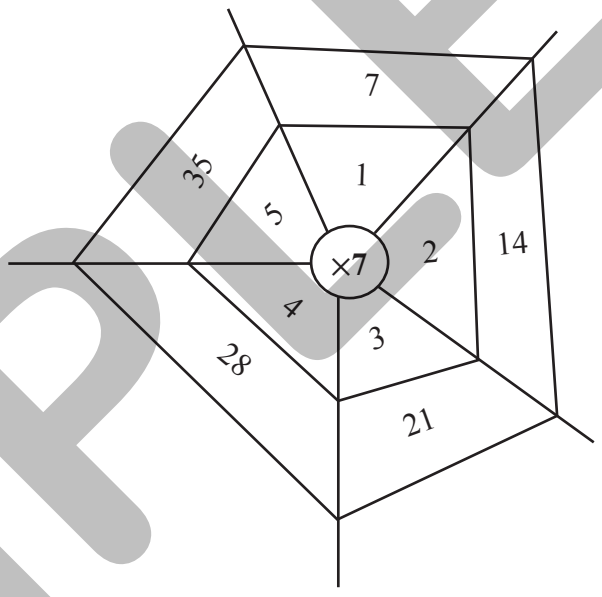
> To find the multiples of a particular number, simply multiply it by the counting numbers (1, 2, 3, 4, etc.).

Example 1:

Say we wished to find the first 5 multiples of 7.

$1 \times 7 = 7$
$2 \times 7 = 14$
$3 \times 7 = 21$
$4 \times 7 = 28$
$5 \times 7 = 35$

OR We can draw a WEB:



The first 5 multiples of 7 are {7, 14, 21, 28, 35}.

Example 2: List all the multiples of 5 which are less than or equal to 30.

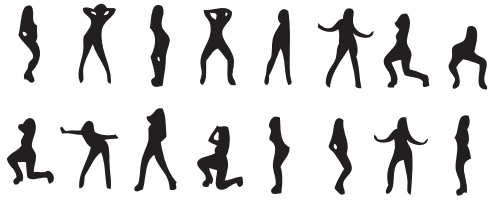
$1 \times 5 = 5$
$2 \times 5 = 10$
$3 \times 5 = 15$
$4 \times 5 = 20$
$5 \times 5 = 25$
$6 \times 5 = 30$

The multiples of 5 less than or equal to 30 are {5, 10, 15, 20, 25, 30}.

Example 3: List all the multiples of 6 which are between 20 and 45.

$1 \times 6 = 6$ $2 \times 6 = 12$ $3 \times 6 = 18$ $4 \times 6 = 24$
$5 \times 6 = 30$ $6 \times 6 = 36$ $7 \times 6 = 42$ $8 \times 6 = 48$

The multiples between 20 and 45 are: {24, 30, 36, 42}.



"Multiple silhouettes"

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LEVEL 1 — NUMBER & PLACE VALUE

EASIER QUESTIONS

Note: Only turn back to page number shown if you have difficulty.

Page

Q1. Write the following numbers in words:
(a) 154 (b) 675 (c) 901 (d) 1 341 — 2, 3

Q2. Write the following numbers as ordinary numerals:
(a) Five hundred & twenty seven (b) Eight hundred & two — 2, 3

Q3. Write the following as ordinary numerals:
(a) $(6 \times 100) + (3 \times 10) + (4 \times 1)$ (b) $(8 \times 100) + (0 \times 10) + (3 \times 1)$ — 2, 3
(c) $(5 \times 1\,000) + (4 \times 100) + (9 \times 10) + (7 \times 1)$

Q4. List all the factors of the following numbers:
(a) 8 (b) 18 (c) 20 (d) 24 (e) 29 — 4

Q5. (a) List the first 5 multiples of 7. (b) List the first 5 multiples of 8. — 5
(c) List the first 8 multiples of 9. (d) List the first 7 multiples of 12.

Q6. State whether the following numbers are prime or composite?
(a) 15 (b) 19 (c) 23 (d) 27 (e) 13 (f) 21 — 10

Q7. List the next 3 numbers in each of the following sequences:
(a) 2, 4, 6, 8, ☐, ☐, ☐ (b) 9, 16, 25, ☐, ☐, ☐ — 9
(c) 1, 3, 6, 10, ☐, ☐, ☐ (d) 7, 9, 11, 13, ☐, ☐, ☐

Q8. Give the name for each set of numbers in Q7. — 9

Q9. (a) Is 74 closer to 70 or 80? Now round off 74 to the nearest 10. — 8
(b) Is 36 closer to 30 or 40? Now round off 36 to the nearest 10.
(c) Is 95 closer to 90 and 100? Now round off 95 to the nearest 10.

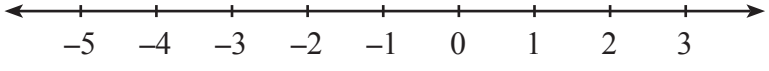
Q10. Find: (a) $\begin{array}{r} 16 \\ +\ 37 \\ \hline \end{array}$ (b) $\begin{array}{r} 43 \\ +\ 21 \\ \hline \end{array}$ (c) $\begin{array}{r} 74 \\ +\ 88 \\ \hline \end{array}$ (d) $\begin{array}{r} 66 \\ +\ 18 \\ \hline \end{array}$ — 12

Q11. Find: (a) $\begin{array}{r} 46 \\ -\ 35 \\ \hline \end{array}$ (b) $\begin{array}{r} 73 \\ -\ 21 \\ \hline \end{array}$ (c) $\begin{array}{r} 56 \\ -\ 16 \\ \hline \end{array}$ (d) $\begin{array}{r} 37 \\ -\ 23 \\ \hline \end{array}$ — 13

Q12. Find: (a) $\begin{array}{r} 13 \\ \times\ 8 \\ \hline \end{array}$ (b) $\begin{array}{r} 17 \\ \times\ 5 \\ \hline \end{array}$ (c) $\begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array}$ (d) $\begin{array}{r} 28 \\ \times\ 3 \\ \hline \end{array}$ — 14

Q13. Find: (a) $8\overline{)24}$ (b) $7\overline{)56}$ (c) $5\overline{)605}$ (d) $9\overline{)639}$ — 15

Q14. By using a solid dot, plot the following numbers on the number line shown:
{–5, 3, 0, –1, –4,}



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Q15. Write the following numbers in ascending order (from smallest to largest):
(a) {4, –7, –5, 0, 2, –9, –3} (b) {–8, 6, –3, 2, 0, –5, 4} — 26, 27

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LEVEL 1 – Number and place value

Q1. (a) One hundred and fifty four (b) Six hundred and seventy five
(c) Nine hundred and one (d) One thousand, three hundred and forty one

Q2. (a) 527 (b) 802

Q3. (a) 634 (b) 803 (c) 5 497

Q4. (a) { 1, 2, 4, 8 } (b) { 1, 2, 3, 6, 9, 18 } (c) { 1, 2, 4, 5, 10, 20 }
(d) { 1, 2, 3, 4, 6, 8, 12, 24 } (e) { 1, 29}

Q5. (a) { 7, 14, 21, 28, 35 } (b) { 8, 16, 24, 32, 40 }
(c) { 9, 18, 27, 36, 45, 54, 63, 72 } (d) { 12, 24, 36, 48, 60, 72, 84 }

Q6. (a) composite (b) prime (c) prime (d) composite (e) prime (f) composite

Q7. (a) 10, 12, 14 (b) 36, 49, 64 (c) 15, 21, 28 (d) 15, 17, 19

Q8. (a) even numbers (b) square numbers (c) triangular numbers (d) odd numbers

Q9. (a) 70 (b) 40 (c) 100

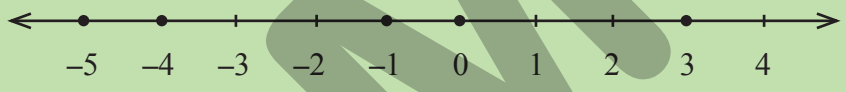
Q10. (a) 53 (b) 64 (c) 162 (d) 84

Q11. (a) 11 (b) 52 (c) 40 (d) 14

Q12. (a) 104 (b) 85 (c) 138 (d) 84

Q13. (a) 3 (b) 8 (c) 121 (d) 71

Q14.



Q15. (a) { –9, –7, –5, –3, 0, 2, 4 } (b) { –8, –5, –3, 0, 2, 4, 6 }

LEVEL 2 – Number and place value

Q1. (a) Three hundred and seventy four (b) Eight hundred and nineteen
(c) Six hundred and two (d) Two thousand, four hundred and fifty seven

Q2. (a) 732 (b) 1 075

Q3. (a) 736 (b) 5 074

Q4. (a) $(3 \times 100) + (2 \times 10) + (5 \times 1)$ (b) $(7 \times 100) + (9 \times 10) + (3 \times 1)$
(c) $(2 \times 100) + (9 \times 10) + (0 \times 1)$ (d) $(9 \times 100) + (0 \times 10) + (7 \times 1)$

Q5. (a) { 1, 2, 3, 4, 6, 12 } (b) { 1, 17 } (c) { 1, 3, 7, 21 }
(d) { 1, 19 } (e) { 1, 3, 9, 27 } (f) {1, 31 }

Q6. (a) composite (b) prime (c) composite (d) prime
(e) composite (f) prime

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