



Accelerated Maths Learning

**UNDERSTANDING
YEAR 9 & 10
M A T H S**

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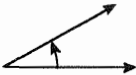
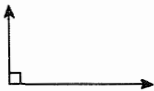
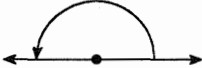
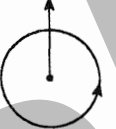
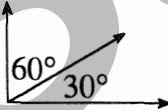
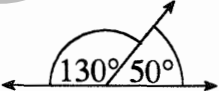
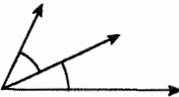
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CHAPTER 3

GEOMETRY

TYPES OF ANGLES

Many of the ideas and theorems presented in this important chapter are only intended to be a brief summary of the work you should have covered in Years 7 and 8.

DIAGRAM	TYPE	ANGLE SIZE
	Acute angle	Less than 90°
	Right angle	Exactly 90°
	Obtuse angle	Between 90° and 180°
	Straight angle	Exactly 180°
	Reflex angle	Between 180° and 360°
	Revolution	Exactly 360°
	Complementary angles	2 angles which add up to 90° or 1 right angle
	Supplementary angles	2 angles which add up to 180°
	Adjacent angles	2 angles with a common vertex which lie on opposite sides of the same arm

THREE IMPORTANT THEOREMS

Geometry is a simple topic providing that you understand and memorise all the new words and facts relating to it. Listed on this page are 3 basic, but important laws, which we call THEOREMS. In later years, you will learn many other geometry theorems.

If angles meet at a point, then their sum must add up to 1 revolution or 360°

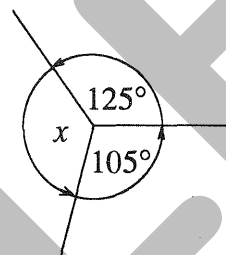
Example:

Find the value of the unknown angle marked x in the diagram shown.

From the theorem above, we know the angle sum = 360° ,

$$\text{i.e. } \hat{x} + 105^\circ + 125^\circ = 360^\circ$$

$$\text{Therefore } \hat{x} = 360^\circ - 105^\circ - 125^\circ = 130^\circ.$$



When a straight angle is divided up into 2 or more angles, then the angle sum must add up to 180°

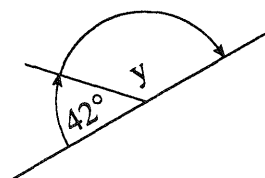
Example:

Find the value of the unknown angle marked y in the diagram shown.

From the theorem above, we know the angle sum = 180° ,

$$\text{i.e. } \hat{y} + 42^\circ = 180^\circ$$

$$\text{Therefore } \hat{y} = 180^\circ - 42^\circ = 138^\circ.$$



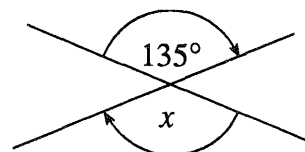
When 2 straight lines intersect (cut each other) then the vertically opposite angles are equal.

Example:

Find the value of the unknown angle marked x in the diagram shown.

From the theorem above, we know that vertically opposite angles are equal.

$$\text{Therefore } \hat{x} = 135^\circ \text{ because it is vertically opposite to } 135^\circ.$$



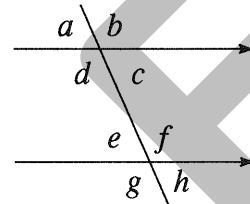
THE PARALLEL LINE THEOREMS

When two parallel lines are cut by a third line, called a TRANSVERSAL, then 8 different angles are formed. There are 3 definite relationships between certain pairs of these angles, and it is important for you to understand the theorems, memorise the words, and know how to apply your knowledge to questions.

CORRESPONDING ANGLES ARE EQUAL

4 pairs of corresponding angles are formed

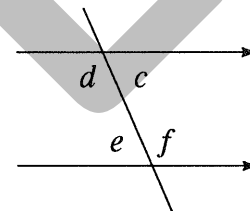
$$\hat{a} = \hat{e}, \quad \hat{b} = \hat{f}, \quad \hat{d} = \hat{g}, \quad \hat{c} = \hat{h}$$



ALTERNATE ANGLES ARE EQUAL

2 pairs of alternate angles are formed

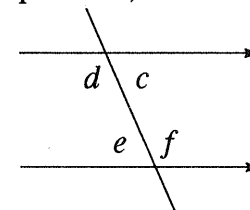
$$\hat{d} = \hat{f} \text{ and } \hat{c} = \hat{e}$$



COINTERIOR ANGLES ARE SUPPLEMENTARY (i.e. add upto 180°)

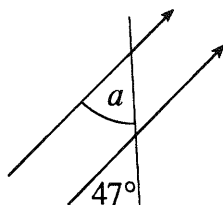
2 pairs of cointerior angles are formed

$$\hat{d} + \hat{e} = 180^\circ \text{ and } \hat{c} + \hat{f} = 180^\circ$$

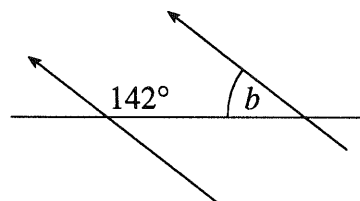


NOTE: In questions relating to the above 3 theorems, the parallel lines are not always horizontal as shown above.

Example: Find the values of the unknown pronumerals in the diagrams below.

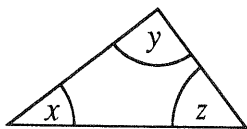


$$\hat{a} = 47^\circ \text{ (corresponding to } 47^\circ \text{)}$$



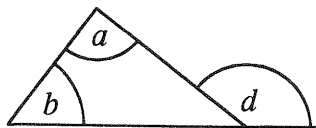
$$\begin{aligned} \hat{b} + 142^\circ &= 180^\circ \text{ (}\hat{b} \text{ is cointerior to } 142^\circ\text{)} \\ \therefore \hat{b} &= 38^\circ \end{aligned}$$

TRIANGLE THEOREMS



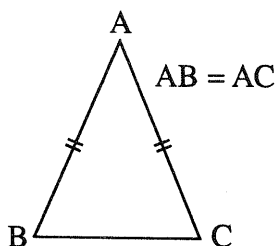
The 3 interior angles of any triangle always add up to 180°

OR $\hat{x} + \hat{y} + \hat{z} = 180^\circ$



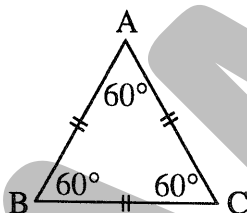
The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

OR $\hat{d} = \hat{a} + \hat{b}$



An ISOSCELES triangle is one which has two equal sides, and the angles at the base of those sides will also be equal.

If $AB = AC$, then $\hat{ABC} = \hat{ACB}$ also.



An EQUILATERAL triangle is one in which all the sides have the same length, and all the angles will be the same size.

i.e. Each angle = 60°

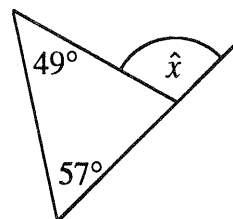
Example:

Find the size of $\hat{x}$ in the diagram shown.

Solution:

From the second theorem above we know that the exterior angle is equal to the sum of the 2 interior opposite angles

$$\begin{aligned}\therefore \hat{x} &= 49^\circ + 57^\circ \\ &= 106^\circ\end{aligned}$$



NOTE: There is a longer way of doing the problem using 2 steps. First calculate the third angle in the triangle which is 74° . Then $\hat{x}$ and 74° are supplementary because they lie on a straight angle. Therefore $\hat{x} = 180^\circ - 74^\circ = 106^\circ$.

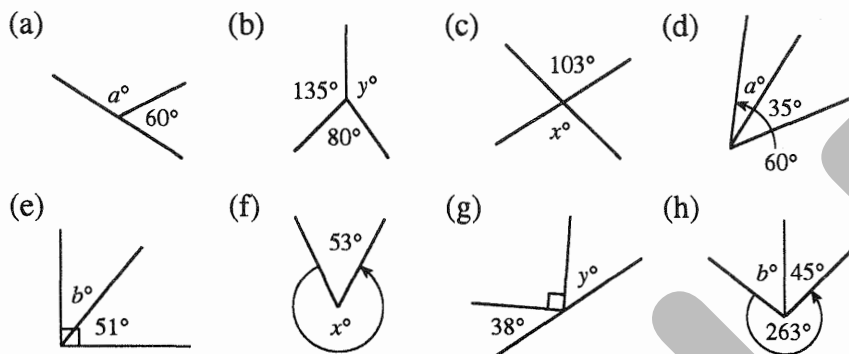
LEVEL 1 — GEOMETRY

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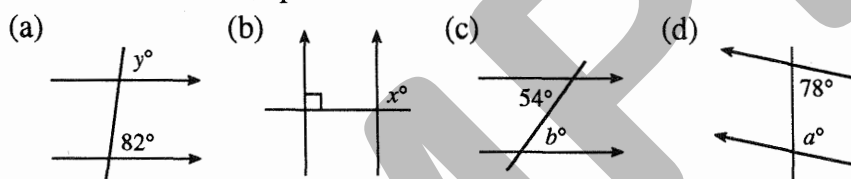
Q1. Write down the value of the pronumeral in each case, giving reasons for your answer:

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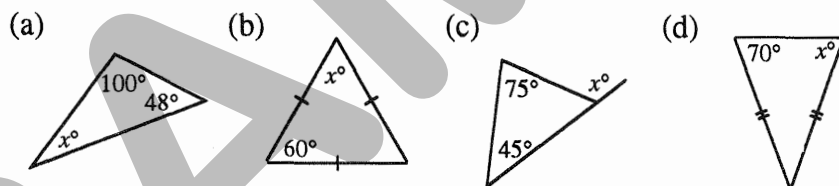
Q2. Find the value of the pronumeral in each case. Give reasons.

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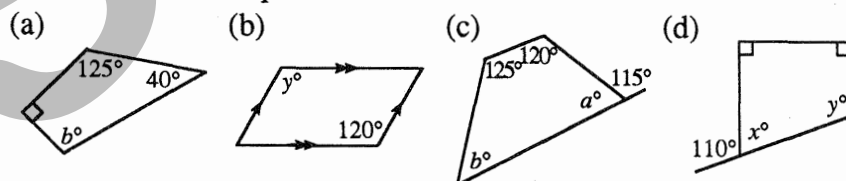
Q3. Find the size of x in the following diagrams. Give reasons.

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Q4. Find the size of the pronumeral in each case. Give reasons.

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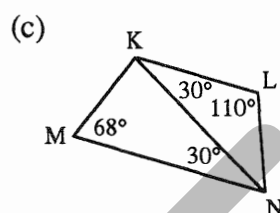
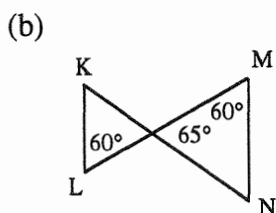
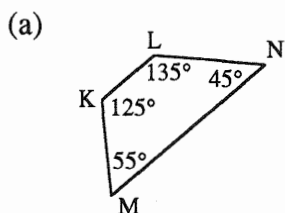
Q5. (a) Find the angle sum of a decagon (i.e. 10 sided figure).

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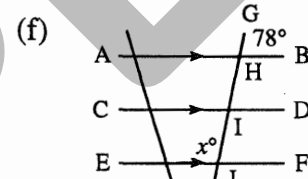
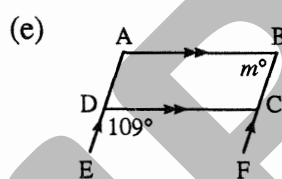
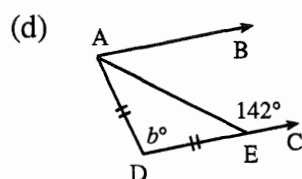
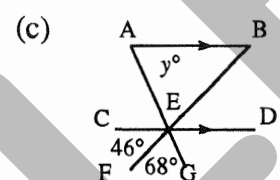
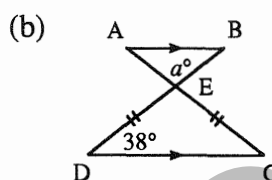
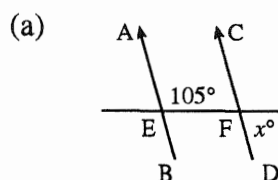
(b) Find the size of each angle in a regular nonagon (i.e. 9 sided figure).

LEVEL 2 — GEOMETRY

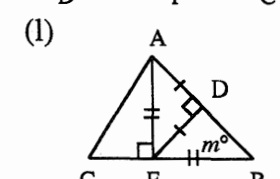
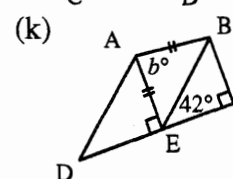
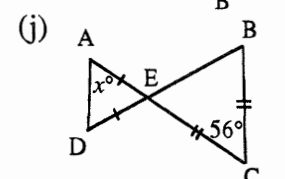
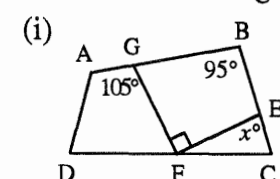
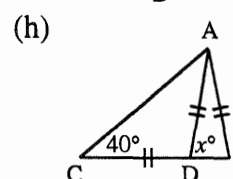
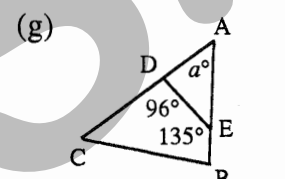
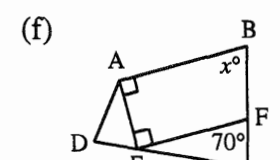
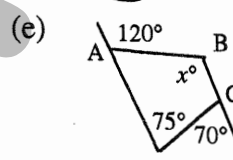
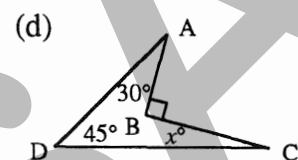
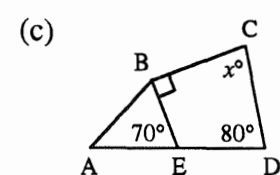
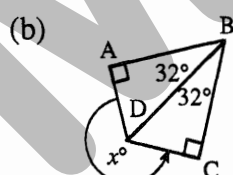
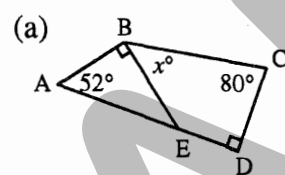
Q1. Give reasons why $KL \parallel MN$ in each of the following:



Q2. Find the value of each pronumeral, giving reasons for your answer:



Q3. Find the value of each pronumeral, giving reasons for your answer:



Level 2 — The Calculator

- Q1. (a) -0.4674 (b) 8540 (c) 9183 (d) 0.003 098 (e) 6.497 (f) 0.2873
 Q2. (a) \$3.13 (b) 8.57 mL (c) 2.83 kg (d) 5.06 km
 (e) \$406.54 (f) 0.97 t
 Q3. (a) \$2362.69 (b) 592.07 mL (c) 13.68 L
 (d) 264.83 kg (e) 1211.11 m (f) \$66.65
 Q4. (a) 4289.877 (b) 1.676 (c) -0.314 (d) 4.354 (e) 3.434 (f) 2.908
 Q5. (a) \$6.90 (b) 153 (c) 0.913 (d) \$1.45
 Q6. (a) -3.14 (b) -18 (c) 10.65 (d) -0.31 (e) 4 (f) 2
 Q7. (a) 9.2×10^3 (b) 5.7×10^{-4} (c) 1.5×10^{-2} (d) 1.2×10^{-6}

Level 1 — Geometry

- Q1. (a) $a = 120^\circ$ (angles on a straight line are supplementary) (b) $y = 145^\circ$ (1 revolution equals 360°)
 (c) $x = 103^\circ$ (vertically opp. angles equal) (d) $a = 60^\circ - 35^\circ = 25^\circ$ ($\hat{a}$ and 35° are adjacent angles)
 (e) $b = 39^\circ$ ($\hat{b}$ is complimentary to 51°) (f) $x = 307^\circ$ (1 revolution equals 360°)
 (g) $y = 52^\circ$ (angles on a straight line are supplementary) (h) $b = 52^\circ$ (1 revolution equals 360°)
- Q2. (a) $y = 82^\circ$ (corresponding angle to 82°) (b) $x = 90^\circ$ (corresponding angle to right angle)
 (c) $b = 54^\circ$ (alternate angle to 54°) (d) $a = 102^\circ$ (cointerior to 78°)
- Q3. (a) $x = 32^\circ$ (angle sum of Δ equals 180°) (b) $x = 60^\circ$ (angle in equilateral Δ)
 (c) $x = 120^\circ$ (exterior angle equals sum of two opposite interior angles) (d) $x = 70^\circ$ (base angle of isosceles Δ)
- Q4. (a) $b = 105^\circ$ (angle sum of quadrilateral equals 360°) (b) $y = 120^\circ$ (opposite angles of parallelogram are equal)
 (c) $a = 65^\circ$ (angles on a straight line are supplementary) (d) $x = 70^\circ$ (angles on a straight line are supplementary)
 $b = 50^\circ$ (angle sum of quadrilateral equals 360°) $y = 110^\circ$ (angle sum of quadrilateral equals 360°)
- Q5. (a) 1440° (b) 140°

Level 2 — Geometry

- Q1. (a) $\angle KLN + \angle LNM = 180^\circ$ ($\therefore$ cointerior) (b) $\angle KLM = \angle LMN$ ($\therefore$ alternate)
 OR $\angle LKM + \angle KMN = 180^\circ$ ($\therefore$ cointerior) $\therefore KL \parallel MN$
 $\therefore KL \parallel MN$
 (c) $\angle LKN = \angle KNM$ ($\therefore$ alternate)
 $\therefore KL \parallel MN$