



Accelerated Maths Learning

ADVANCED COURSE

UNDERSTANDING

YEAR 9 & 10

M A T H S

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CONTENTS

Preface(x)

Features and benefits.....(xi)

How to use this book effectively(xii)

1. ALGEBRA

1

Introduction — substitution — addition & subtraction — multiplication and division — the four basic operations on algebraic fractions — removing grouping symbols — expanding binomial products — special binomial products.

3 topic review exercises

2. INDICES

17

Index notation — the basic laws — fractional and negative indices — typical exam questions — scientific notation for large and small numbers.

2 topic review exercises

3. MEASUREMENT

25

Important units and conversions — perimeter and areas of plane and composite shapes — surface areas and volumes of prisms and cylinders.

2 topic review exercises

4. CONSUMER ARITHMETIC

37

Ways of earning money — overtime rates — allowances and other important terms — income tax — spending money.

2 topic review exercises

5. EQUATIONS

45

Introduction — inverse operations — one step equations — equations involving brackets and fractions — checking the answer — inequations and the number line — applications of equations — substituting into formulae — changing the subject of the formula — constructing formulae.

3 topic review exercises

6. SURDS

63

Pythagoras' theorem — irrational numbers — multiplication and division of surds — simplifying surds — adding and subtracting surds — expansion of brackets — special binomial products and conjugate surds — rationalising one term denominators and binomial denominators.

2 topic review exercises

7. COORDINATE GEOMETRY

75

Ordered pairs — midpoint formula — distance between two points — gradient of a line — graphing straight lines — special line graphs — gradient/intercept form of a line — sketching lines — general form of a line — point/gradient formula — two point formula — parallel lines — perpendicular lines — collinear points and concurrent lines.

2 topic review exercises

8. GRAPHS

95

Introduction — summary of graphs from Year 8 — column graphs — bar graphs — sector graphs — line graphs — step graphs — travel graphs — line of best fit — matching graphs in practical situations.

2 topic review exercises

9. STATISTICS

105

Frequency distribution tables — histograms — mean, median, mode and range — finding the mean from a distribution table — grouped frequency table — cumulative frequency — finding the median from a cumulative frequency histogram — statistics and the calculator.

2 topic review exercises

10. SIMULTANEOUS EQUATIONS **117**

Graphical methods — substitution method — elimination method — problem solving applications.

1 topic review exercise

11. FURTHER ALGEBRA AND FACTORISATION **123**

Factorising binomials — trinomials — four term expressions — harder trinomials — the reasons for factorising — multiplying and dividing algebraic fractions — adding and subtracting algebraic fractions.

2 topic review exercises

12. GEOMETRY **133**

Types of angles — basic theorems — parallel line theorems — triangle theorems — quadrilaterals — polygons — reasoning in geometry — general proofs — congruent triangles — setting out proofs — using congruency to prove other facts — constructions in geometry.

2 topic review exercises

13. TRIGONOMETRY **151**

Naming the sides — the calculator — sine, cosine and tangent ratios — unknown in the denominator — angles of elevation and depression — bearings — more difficult applications requiring two stages.

2 topic review exercises

14. FURTHER CONSUMER ARITHMETIC **165**

Simple interest — compound interest — compound interest formula — depreciation — purchasing by instalments — examples on time payment.

1 topic review exercise

15. QUADRATIC EQUATIONS **173**

Solving by factorisation — completing the square — the quadratic formula — applications of quadratic equations.

1 topic review exercise

16. SURFACE AREA AND VOLUME **181**

Surface area of prisms and other solids — volumes of prisms, cones, pyramids and spheres — composite volumes.

2 topic review exercises

17. GRAPHS **189**

The straight line — sketching parabolas — graphing regions — the cubic curve — the hyperbola — the exponential curve — the circle — summary of curve features.

2 topic review exercises

18. SIMULTANEOUS EQUATIONS & GRAPHS **203**

Intersection of 2 straight lines — using simultaneous equations to find intersection of lines and parabolas, lines and hyperbolas.

2 topic review exercises

19. FURTHER STATISTICS **209**

Relative frequency — range — standard deviation using both the formula and the calculator — the normal curve.

1 topic review exercise

20. PROBABILITY **215**

The basic law — range of probabilities — tree diagrams — probabilities involving coins and dice — replacement and non replacement problems — multiplying outcomes and counting techniques.

1 topic review exercise

21. SIMILARITY **225**

Introduction — similar triangles — areas of similar figures — volumes of similar figures.

1 topic review exercise

22. FURTHER TRIGONOMETRY **233**

The sine rule — the cosine rule — the cosine rule transposed — area of a triangle — angles larger than 90° — special angles — more difficult applications.

2 topic review exercise

LOBES

23. GEOMETRY OF THE CIRCLE **245**

Important parts of a circle — angle theorems and applications — chord theorems and applications — tangent theorems and applications — alternate angle theorem — intersecting chords — secants and tangents.

2 topic review exercises

24. LOGARITHMS **255**

Revision of index laws — exponential equations — base 10 logarithms — the general definition — change of base law — the 5 other laws relating to logarithms — applications — more difficult exponential equations — graphs of log and exponential functions.

2 topic review exercises

25. POLYNOMIALS **269**

Introduction and terminology — addition, subtraction, multiplication and division of polynomials — graphs of basic functions — the remainder theorem — the factor theorem — sketching polynomials and other applications of the factor theorem.

2 topic review exercises

SOLUTIONS **283**

CHAPTER 1

ALGEBRA

INTRODUCTION

You should already be familiar with many of the ideas explained in this chapter from earlier work covered in Years 7 and 8. Algebra is a very important branch of Mathematics which involves the study of ‘pronumerals’ — i.e. letters which can represent numbers in an expression. Algebra is used in many other topics in Maths (e.g. index laws, equations, formulae, etc.) and therefore it is essential that you obtain a thorough understanding of this initial chapter.

Algebraic expressions (which involve pronumerals and numbers) are formed using exactly the same steps as arithmetic expressions (which involve numbers only). Therefore algebra is a generalised extension of simple arithmetic.

In each example below, the steps to the solution of a problem are firstly shown using ordinary numerals. These steps are then applied in exactly the same way using pronumerals.

Example: The average of 10 and 6 = $\frac{10+6}{2}$
 $\therefore$ The average of x and y = $\frac{x+y}{2}$

Example: The cost of 5 bananas at 40 c each = (5×40) cents
 $\therefore$ The cost of p bananas at 40 c each = $p \times 40$
 $= 40p$ cents

Example: If I walk 10 km in 2 hours my speed = $\frac{10}{2}$ km/h
 $\therefore$ If I walk m km in n hours my speed = $\frac{m}{n}$ km/h

Example: A man now aged 30 years, will in 7 years = $(30 + 7)$ years old
A man now aged x years, will in 7 years = $(x + 7)$ years old

Example: The next two odd numbers after 11 are $11 + 2$, $11 + 4$
The next two odd numbers after odd y are $y + 2$, $y + 4$

SUBSTITUTION

Substitution has been covered in previous years, and this section is only meant to be a brief review of what you should already know. Always keep in mind that 'pronumerals' or 'letters' in algebra represent numbers.

If the numerical value of each letter is given to you in an algebraic expression, then it is a simple process to substitute in the numbers for the corresponding letters, and hence find the value of the expression.

Example: If $a = 3, b = -4, c = 2, d = \frac{1}{2}$ find the value of the following expressions:

- (i) $3a + 2b$ (ii) $b^2 + d^2$ (iii) $\sqrt{5a - b + 3c}$ (iv) $\frac{c - b}{d}$
(v) $abcd$ (vi) $d(b - c)$ (vii) $\sqrt[3]{bc}$ (viii) $3b^2 \div ac$

Solutions:

$$\begin{aligned}\text{(i)} \quad 3a + 2b &= 3 \times 3 + 2 \times -4 \\ &= 9 + (-8) \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad \sqrt{5a - b + 3c} &= \sqrt{5 \times 3 - (-4) + 3 \times 2} \\ &= \sqrt{15 + 4 + 6} \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{(v)} \quad abcd &= 3 \times -4 \times 2 \times \frac{1}{2} \\ &= -12\end{aligned}$$

$$\begin{aligned}\text{(vii)} \quad \sqrt[3]{bc} &= \sqrt[3]{-4 \times 2} \\ &= \sqrt[3]{-8} \\ &= -2\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad b^2 + d^2 &= (-4)^2 + \left(\frac{1}{2}\right)^2 \\ &= 16 + \frac{1}{4} \\ &= 16\frac{1}{4}\end{aligned}$$

$$\begin{aligned}\text{(iv)} \quad \frac{c - b}{d} &= \frac{2 - (-4)}{\frac{1}{2}} \\ &= 6 \div \frac{1}{2} \\ &= 12\end{aligned}$$

$$\begin{aligned}\text{(vi)} \quad d(b - c) &= \frac{1}{2}(-4 - 2) \\ &= -3\end{aligned}$$

$$\begin{aligned}\text{(viii)} \quad 3b^2 \div ac &= 3 \times (-4)^2 \div 3 \times 2 \\ &= 48 \div 3 \times 2 \\ &= 32\end{aligned}$$

ADDING AND SUBTRACTING IN ALGEBRA

“LIKE TERMS” in algebra are letters, or groups of letters, which are identical except for the leading number in front.

For Example:

$5a, -2a, 7a$	are all like terms
$-3ab, 6ab, 2ab$	are all like terms
$8x^2, 3x^2, -x^2$	are all like terms

“UNLIKE TERMS” contain different letters or groups of letters

For Example:

$7a, 3b, -4ab$	are all unlike terms
$5c, -6d, 2ac$	are all unlike terms
$4x^2, -3x, 2$	are all unlike terms

In algebra, one can only add or subtract like terms

Example: Simplify $8x^2 - 3x + 2x^2 + 5x - 4$

$$\begin{array}{|c|} \hline + 8x^2 \\ \hline \end{array} \begin{array}{|c|} \hline - 3x \\ \hline \end{array} \begin{array}{|c|} \hline + 2x^2 \\ \hline \end{array} \begin{array}{|c|} \hline + 5x \\ \hline \end{array} \begin{array}{|c|} \hline - 4 \\ \hline \end{array} = 10x^2 + 2x - 4$$

$+ 8x^2$ and $+ 2x^2$ are like terms

$- 3x$ and $+ 5x$ are like terms



Examples: Simplify the following expressions by adding or subtracting like terms:

- (i) $2x + 3y + 5x + y = 7x + 4y$
- (ii) $6m - 3n + 2m + 5n = 8m + 2n$
- (iii) $6a - 3b - 2a - 6b = 4a - 9b$
- (iv) $8ab + 5b - 10ab - 2b = -2ab + 3b$
- (v) $5x^2 - 7x + x^2 + 4x - 8 = 6x^2 - 3x - 8$

NOTE:

The sign always belongs to the term directly after it.

MULTIPLYING AND DIVIDING PRONUMERALS

The main differences between algebra and ordinary numerals occur in the way multiplication and division is written. In algebra, the multiplication sign is left out. Also with division, the sign is usually left out and the expression is written in fraction form.

REMEMBER:

In multiplication $p \times q$ is written pq .

In division, $p \div q$ is written $\frac{p}{q}$.

Examples: Simplify the following expressions

(i) $-8 \times 2m = -16m$

Negative times positive = negative

(ii) $a \times b \times 5 \times a = 5a^2b$

$a \times a = a^2$. Number is always written in the front.

(iii) $-12m \div 6 = \frac{-12m}{6} = -2m$

Negative $\div$ positive = negative

(iv) $\frac{6ab}{8a} = \frac{3b}{4}$

Divide top & bottom by $2a$.

(v) $\frac{15m^2}{5m} = 3m$

Divide top & bottom by $5m$.

(vi) $(-3a) \times (-2a) \times (-4a) = -24a^3$

$-3 \times -2 \times -4 = -24$ $a \times a \times a = a^3$

(vii) $\frac{(4ab)^2}{8ab^2} = \frac{16a^2b^2}{8ab^2} = 2a$

$(4ab)^2 = 4ab \times 4ab$

NOTE: Simplifying algebraic fractions is done in the same way as ordinary fractions, by dividing the numerator and the denominator by the highest common factor.

ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

In earlier sections, you have already learned how to perform the 4 operations (+ − × ÷) on simple algebraic expressions. Now it is necessary to know how to perform the 4 operations (+ − × ÷) on ALGEBRAIC FRACTIONS.

The steps required are exactly the same as with ordinary fractions.

- (i) Where necessary, rewrite each fraction with a common denominator.
- (ii) Then add or subtract the numerators.
- (iii) Where possible, simplify the resulting expression by cancelling down.

Examples:

Simplify the following expressions:

$$(i) \quad \frac{11}{3x} - \frac{5}{3x} = \frac{11-5}{3x}$$

Already has a common denominator.

$$= \frac{6}{3x}$$

Subtract the numerators.

$$= \frac{2}{x}$$

Cancels down.

$$(ii) \quad \frac{3m}{7} + \frac{4m}{5} = \frac{15m}{35} + \frac{28m}{35}$$

Lowest common denominator = 35.

$$= \frac{43m}{35}$$

Add the numerators.

$$(iii) \quad \frac{4a}{3b} - \frac{2a}{5b} = \frac{20a}{15b} - \frac{6a}{15b}$$

Lowest common denominator = 15b.

$$= \frac{14a}{15b}$$

Subtract numerators.

$$(iv) \quad \frac{x+1}{4} - \frac{2x}{8} = \frac{2(x+1)}{8} - \frac{2x}{8}$$

Lowest common denominator = 8.

$$= \frac{2x+2-2x}{8}$$

$$\frac{x+1}{4} = \frac{2(x+1)}{8} = \frac{2x+2}{8}$$

$$= \frac{2}{8}$$

Expand brackets & simplify.

$$= \frac{1}{4}$$

MULTIPLYING ALGEBRAIC FRACTIONS

The steps for multiplying algebraic fractions are exactly the same as for ordinary numerical fractions.

- (i) Multiply out the numerators.
- (ii) Multiply out the denominators.
- (iii) Simplify where possible by cancelling down.

Examples:

Simplify the following:

(i)
$$\frac{m}{3} \times \frac{2}{x} = \frac{m \times 2}{3 \times x}$$
$$= \frac{2m}{3x}$$

$m \times 2$ is written as $2m$

(ii)
$$\frac{3}{x} \times \frac{x}{2} = \frac{3x}{2x}$$
$$= \frac{3}{2}$$

The x 's cancel each other out to leave $\frac{3}{2}$

(iii)
$$\frac{5n}{2} \times \frac{4m}{15} = \frac{20mn}{30}$$
$$= \frac{2mn}{3}$$

$\frac{20}{30}$ simplifies to $\frac{2}{3}$

(iv)
$$\frac{8a}{b} \times \frac{3b}{5} \times \frac{4c}{3a} = \frac{96abc}{15ab}$$
$$= \frac{32c}{5}$$

The ' ab ' cancel each other

$\frac{96}{15}$ simplifies to $\frac{32}{5}$

(v)
$$\frac{3x^2y}{2a} \times \frac{8a^3}{5xy} = \frac{24x^2ya^3}{10axy}$$
$$= \frac{12xa^2}{5}$$

$\frac{24}{10}$ simplifies to $\frac{12}{5}$

$\frac{x^2}{x} = x$, $\frac{a^3}{a} = a^2$, $\frac{y}{y} = 1$

DIVIDING ALGEBRAIC FRACTIONS

Once again, the steps for dividing algebraic fractions are exactly the same as for ordinary numerical fractions covered in Year 7.

- (i) Change the division sign to a multiply sign.
- (ii) Turn the second fraction upside down.
- (iii) Now multiply as shown on the opposite page.

These steps are usually remembered by students as

“INVERT AND MULTIPLY”.

Examples:

Simplify the following:

(i) $\frac{m}{2} \div \frac{n}{6} = \frac{m}{2} \times \frac{6}{n}$ Invert and multiply.

$$= \frac{6m}{2n}$$

$\frac{6}{2}$ simplifies to 3

$$= \frac{3m}{n}$$

(ii) $\frac{5a}{4} \div \frac{a}{8} = \frac{5a}{4} \times \frac{8}{a}$ Invert and multiply.

$$= \frac{40a}{4a}$$

The a 's cancel. Also $\frac{40}{4} = 10$

$$= 10$$

(iii) $\frac{ab}{3} \div \frac{a}{b} = \frac{ab}{3} \times \frac{b}{a}$ Invert and multiply.

$$= \frac{ab^2}{3a}$$

The a 's cancel.

$$= \frac{b^2}{3}$$

(iv) $\frac{6x^2}{5y} \div \frac{3x}{10y^3} = \frac{6x^2}{5y} \times \frac{10y^3}{3x}$ Invert and multiply.

$$= \frac{60x^2y^3}{15xy}$$

$\frac{x^2}{x}$ simplifies to x

$$= 4xy^2$$

$\frac{y^3}{y^1}$ simplifies to y^2

REMOVING GROUPING SYMBOLS

“Grouping symbols” means exactly the same as “BRACKETS”. You will often be asked to remove grouping symbols from a particular algebraic expression.

This is done by multiplying the term outside the brackets by each of the terms inside the brackets.

The word “**EXPAND**” is often used instead of “remove grouping symbols”.

THE GENERAL RULES

$$a(b + c) = a \times b + a \times c = ab + ac$$

$$a(b - c) = a \times b - a \times c = ab - ac$$

Examples:

Expand the following:

$$\begin{aligned} \text{(i)} \quad 6(a - 3) &= 6 \times a - 6 \times 3 \\ &= 6a - 18 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 4m(m + 5) &= 4m \times m + 4m \times 5 \\ &= 4m^2 + 20m \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad -5(p + 2) &= -5 \times p + (-5) \times 2 \\ &= -5p - 10 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad -3(2a - 5) &= -3 \times 2a + (-3) \times (-5) \\ &= -6a + 15 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad -(2 + 3y) &= -1(2 + 3y) \\ &= -1 \times 2 + (-1) \times (3y) \\ &= -2 - 3y \end{aligned}$$

Place a 1 outside the bracket if there is no numbers.

The last three examples are important to understand because they occur regularly in algebra.

A minus sign outside of a bracket reverses the sign inside of the bracket.

MORE DIFFICULT EXAMPLES

In the examples on this page, it is first necessary to expand the brackets in the usual way. Then the resulting expressions are simplified by adding or subtracting the like terms.

Examples: Remove grouping symbols from the following, and simplify the resulting expressions:

$$\begin{aligned}\text{(i)} \quad & 5(x + 3) + 4(x - 2) \\ & = 5x + 15 + 4x - 8 \\ & = 9x + 7\end{aligned}$$

Remove brackets

Add like terms

$$\begin{aligned}\text{(ii)} \quad & 4(2a - 3) + 3(a + 2) \\ & = 8a - 12 + 3a + 6 \\ & = 11a - 6\end{aligned}$$

Remove brackets

Add like terms

$$\begin{aligned}\text{(iii)} \quad & x^2(2x - 5) + 3x(4 - 5x) \\ & = 2x^3 - 5x^2 + 12x - 15x^2 \\ & = 2x^3 - 20x^2 + 12x\end{aligned}$$

Remove brackets

Add like terms

$$\begin{aligned}\text{(iv)} \quad & 5(2m + 4) - 3(m + 5) \\ & = 10m + 20 - 3m - 15 \\ & = 7m + 5\end{aligned}$$

Remove brackets

Note: $-3 \times +5 = -15$

$$\begin{aligned}\text{(v)} \quad & 6(p - 5) - 4(p - 3) \\ & = 6p - 30 - 4p + 12 \\ & = 2p - 18\end{aligned}$$

Remove brackets

Note: $-4 \times -3 = +12$

$$\begin{aligned}\text{(vi)} \quad & 3x(2x - 4) - (3 - 2x) \\ & = 3x(2x - 4) - 1(3 - 2x) \\ & = 6x^2 - 12x - 3 + 2x \\ & = 6x^2 - 10x - 3\end{aligned}$$

Place a 1 outside $-(3 - 2x)$

Remove brackets

$-1 \times -2x = +2x$

NOTE: Once again, watch the signs carefully when there is a minus sign outside of the bracket, as shown in the last two examples. When the brackets are removed, the minus sign outside the bracket has reversed the sign inside the bracket.

LEVEL 1 — ALGEBRA

Note: Only turn back to page number if you have difficulty

Page

- Q1. (a) A car travels y km/h for x hours. How far does it travel?
 (b) Two angles of a triangle are x° and y° . What is the size of the third angle?
 (c) Three people have \$ x , \$ y and \$ z respectively. What is their average wealth?

1

Q2. If $p = 6$, $q = 2$, $r = 4$ and $s = 3$, find:

2

- (a) $ps + qr$ (b) $pqrs$ (c) $\sqrt{2p + r}$
 (d) $2(p + q) - r^2$ (e) $5r + 2q - 3s$ (f) $\frac{1}{2}pq + \frac{1}{3}rs$

Q3. Simplify:

3

- (a) $5x + 3x^2 - 2x + 6x^2$ (b) $p^2 + 2p - 5 - 6p + 3p^2$
 (c) $6xy - 5x + 4y - 3xy$ (d) $x^2 + 3y + 3xy - x^2 + 5xy$

Q4. Simplify:

4

- (a) $5a \times b \times 3c$ (b) $6p \times 4q \times \frac{1}{2}p$ (c) $3xy \times -5x$
 (d) $16mn \div 2n$ (e) $4x^2y \div 2x$ (f) $24abc \div 2ac$

Q5. Simplify:

5

- (a) $\frac{m}{2} + \frac{m}{4}$ (b) $\frac{x}{3} - \frac{x}{4}$ (c) $\frac{2a}{5} + \frac{a}{2}$
 (d) $\frac{a}{x} + \frac{b}{x}$ (e) $\frac{3x}{2} - \frac{2y}{3}$ (f) $\frac{m}{3} - \frac{2m}{7}$

Q6. Simplify:

6, 7

- (a) $\frac{x}{2} \times \frac{x}{4}$ (b) $\frac{5}{a} \times \frac{b}{10}$ (c) $\frac{4}{m} \times \frac{3}{2n}$
 (d) $\frac{a}{3} \div \frac{a}{4}$ (e) $\frac{2x}{3} \div \frac{x}{6}$ (f) $\frac{4}{m} \div \frac{3n}{m}$

Q7. Expand the following:

8, 9

- (a) $5(x + 2)$ (b) $3(2a - 1)$ (c) $2y(y + 3)$
 (d) $4(3 - x)$ (e) $-2(m + n)$ (f) $b(2 - 3b)$

Q8. Expand these binomial products:

10, 11

- (a) $(x + 2)(x + 1)$ (b) $(x - 1)(x - 3)$ (c) $(m + 3)(m - 4)$
 (d) $(a + 3)(b - 2)$ (e) $(m - 9)(n + 2)$ (f) $(b - 2)(a + 2)$

LEVEL 1 — POLYNOMIALS

Note: Only turn back to page number if you have difficulty

Page

Q1. Which of the following expressions are not polynomials: (a) $2x^5 + 5x^2 + 9$ (b) $3x^4 + (x-3)^2$ (c) $x^4 + 3x - x^2$ (d) $x^3 + 2x^{\frac{1}{2}} - 3x$ (e) $\sqrt{5}x^5 + 3x^2 + 10$ (f) $3x^2 + 2x^{-1} + 5$	269
Q2. For each polynomial state the degree, leading coefficient and constant term: (a) $5x^3 + 9x - 2x^2 + 5$ (b) $6x^2 - x^3 - 5$ (c) $15x^4 - 3x^5 + 2x^2 - 4x$ (d) $8 - x^2 + 3x - x^6$ (e) $9x^5 - 4x^2$ (f) $7x^5 - 2x^2 + 5x^3 - 6$	269
Q3. For each polynomial function find $P(-1)$, $P(1)$ and $P(2)$: (a) $P(x) = 4x^2 + 3x - 7$ (b) $P(x) = 16 - 3x$ (c) $P(x) = x^3 - 3x + 5x^2$ (d) $P(x) = 6x^4 - 5x^3 - 2x$ (e) $P(x) = 5x^5 + 1$ (f) $P(x) = 1 - 4x^4 + x^2$	269
Q4. If $P(x) = 2x^2 - 3x + 1$, $Q(x) = x^3 + 2x$ and $R(x) = 2x^3 - 5$, find: (a) $P(x) + Q(x)$ (b) $P(x) - R(x)$ (c) $P(x) + Q(x) + R(x)$ (d) $P(x) - Q(x) - R(x)$ (e) $P(x) \cdot Q(x)$ (f) $Q(x) \cdot R(x)$	270
Q5. Sketch the following polynomials: (a) $y = (x+3)(x-1)(x-2)$ (b) $y = (x-3)(x+1)(4-x)$ (c) $y = x(x-3)^2$ (d) $y = (x+4)(x-2)(x+1)^2$ (e) $y = x(x+5)(x-1)(x-3)$ (f) $y = -x^2(x-3)(x+3)$	272
Q6. Complete the divisions below and express them in the form $P(x) = A(x)Q(x) + R(x)$: (a) $(5x^2 + 15x + 8) \div (x+3)$ (b) $(6x^2 - 9x + 10) \div (x-1)$ (c) $(2x^3 + 3x^2 + 5x - 4) \div (2x+1)$ (d) $(7x^3 + 3x - 3) \div (x+1)$	273, 274
Q7. Use the remainder theorem to find the remainder in each case: (a) $(3x^2 + 5x + 2) \div (x+2)$ (b) $(x^3 - 2x^2 + 4x + 6) \div (x-3)$ (c) $(2x^4 - 5x^2 + 10x - 5) \div (x-1)$ (d) $(x^4 + 3x^3 + 2x - 4) \div (x+4)$	275
Q8. Find the factors of the following polynomials: (a) $x^3 - 7x - 6$ (b) $x^3 + 2x^2 - x - 2$ (c) $3x^3 - 4x^2 - 35x + 12$ (d) $2x^3 + 3x^2 - 11x - 6$ (e) $2x^3 - x^2 - 32x + 16$ (f) $x^3 + 3x^2 + 7x + 10$	276, 277
Q9. Find the real roots of these equations: (a) $x^3 - x^2 - x + 1 = 0$ (b) $x^3 - 7x - 6 = 0$ (c) $x^3 - 4x^2 + 4x = 0$	278