

Accelerated Maths Learning

# EXTENSION 1 (2&3 UNIT COURSE)

# UNDERSTANDING YEAR 11 MATHS

SAMPLE

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# CHAPTER 3

# FACTORISING


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# FACTORISING BINOMIALS

Algebraic expressions which contain only two terms are called 'binomials'. Factorisation involves the reverse procedure of removing or expanding brackets—an idea which has been fully covered in Chapter 2.

If we expand $a(b+c)$, we obtain $ab+ac$.

If we factorise $ab+ac$, we obtain $a(b+c)$.

In any factorisation, the very first step is to always:

> Look for the highest common factor (HCF) and write it outside the bracket.

With binomials, then look for the 'Difference of two squares' pattern $(\quad)^2-(\quad)^2$, and factorise according to the formula below:

$$a^2-b^2=(a+b)(a-b)$$

Examples:

(i) $9a^2+12ab = 3a(3a+4b)$ HCF $= 3a$

(ii) $-4x^2+20x = -4x(x-5)$ HCF $= -4x$

(iii) $-5m^3-10m^2 = -5m^2(m+2)$ HCF $= -5m$

(iv) $16x^2-25 = (4x)^2-5^2$ $a^2-b^2$ pattern
$= (4x+5)(4x-5)$

(v) $4m^2-16 = 4(m^2-4)$ HCF $= 4$ and then
$= 4(m+2)(m-2)$ $a^2-b^2$ pattern

(vi) $x^4-y^4 = (x^2+y^2)(x^2-y^2)$ $a^2-b^2$ pattern
$= (x^2+y^2)(x+y)(x-y)$ $x^2-y^2$ can be further factorised

*Note* We cannot factorise $a^2+b^2$.

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## SUM AND DIFFERENCE OF TWO CUBES

This factorisation technique is sometimes asked in test questions, and it follows the same principle as the difference of two squares. The student should look out for the pattern:

$$(\quad)^3 + (\quad)^3$$
$$\text{or } (\quad)^3 - (\quad)^3$$

The following two techniques, or formulae, are used to factorise algebraic expressions of this type:

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$
$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

Examples:

(i) $x^3+27$

$= x^3+3^3$ — $a^3+b^3$ pattern

$= (x+3)(x^2-3x+3^2)$ — $(a+b)(a^2-ab+b^2)$

$= (x+3)(x^2-3x+9)$

(ii) $8m^3-125$

$= (2m)^3-5^3$ — $a^3-b^3$ pattern

$= (2m-5)\left[\,(2m)^2+2m\times5+5^2\,\right]$ — $(a-b)(a^2+ab+b^2)$

$= (2m-5)(4m^2+10m+25)$

(iii) $6x^3-48$

$= 6(x^3-8)$ — First take out HCF = 6

$= 6(x^3-2^3)$ — $a^3-b^3$ pattern

$= 6(x-2)(x^2+2x+4)$ — $(a-b)(a^2+ab+b^2)$

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## FOUR-TERM EXPRESSIONS

These might initially sound complicated, but in reality they are easier to factorise than quadratic expressions. They must be understood, not only on their own account, but also in order to factorise harder quadratics in the next section.

The steps, in order, are outlined below:

| | |
|---|---|
| STEP 1 | Split into two groups, each containing two terms. |
| STEP 2 | Factorise each group separately. |
| STEP 3 | Both brackets should be the same. |
| STEP 4 | Factorise again. |

Examples:

(i) Factorise $x^2+3x+bx+3b$.

| | |
|---|---|
| $\boxed{x^2+3x} + \boxed{bx+3b}$ | Split into two groups |
| $= x(x+3)+b(x+3)$ | Factorise each group separately |
| $= (x+b)(x+3)$ | Factorise again |

(ii) Factorise $x^2-2x-5x+10$.

| | |
|---|---|
| $\boxed{x^2-2x} - \boxed{5x+10}$ | Split into two groups |
| $= x(x-2)-5(x-2)$ | Factorise each group separately |
| $= (x-5)(x-2)$ | Factorise again |

*Note* carefully above that $-5x+10 = -5(x-2)$.

(iii) Factorise $a^3+a^2-a-1$.

| | |
|---|---|
| $\boxed{a^3+a^2} - \boxed{a-1}$ | Split into two groups |
| $= a^2(a+1)-1(a+1)$ | Factorise each group separately |
| $= (a^2-1)(a+1)$ | Factorise again |
| $= (a+1)(a-1)(a+1)$ | $a^2-1$ can be further factorised |
| $= (a+1)^2(a-1)$ | because it has pattern $a^2-b^2$. |

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# QUADRATIC FACTORS—METHOD 1

These are algebraic expressions which contain three terms expressed in the format $ax^2+bx+c$

In this section we shall first consider the factorisation of easier trinomials where the leading coefficient $a = 1$.

**To factorise $x^2+bx+c$:**

STEP 1 Ask what two numbers multiply together to give $c$ and add to give $b$.

STEP 2 Let the two numbers be $\alpha$ and $\beta$.

STEP 3 Write the factors as $(x+\alpha)(x+\beta)$.

Examples:

(i) Factorise $x^2-2x-8$

What two numbers multiply to give $-8$ and add to give $-2$?

That is, $\alpha\beta = -8$ and $\alpha+\beta = -2$.

We can see that $\alpha = -4$ and $\beta = +2$,

therefore the factors of $x^2-2x-8$ are $(x-4)(x+2)$.

(ii) Factorise $x^2-x-12$

What two numbers multiply to give $-12$, and add up to give $-1$?

That is, $\alpha\beta = -12$ and $\alpha+\beta = -1$.

We can see that $\alpha = -4$ and $\beta = +3$,

therefore the factors of $x^2-x-12$ are $(x-4)(x+3)$.

(iii) Factorise $12+x-x^2$

Firstly use the '–1' trick and write $12+x-x^2$ as $-(-12-x+x^2)$,

that is, factorise $-(x^2-x-12) = -(x-4)(x+3)$

$= (4-x)(x+3)$

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# QUADRATIC FACTORS—METHOD 2

Many of the quadratics given in test questions do not have a leading coefficient of $a = 1$. These types are more difficult to factorise and the following steps are recommended:

**To factorise** $ax^2+bx+c$, where $a \neq 1$:

STEP 1 Multiply $a$ and $c$.

STEP 2 Find two numbers $\alpha$ and $\beta$ that multiply to give the product $ac$ and add up to give $b$.

STEP 3 Split the original quadratic into a four-term expression: $ax^2+\alpha x+\beta x+c$

STEP 4 Factorise the four-term expression using the steps on Page 32.

Examples:

(i) Factorise $3x^2+16x-12$.

Multiply 3 by $-12$ to give $-36$. Think of two numbers $\alpha$ and $\beta$ which multiply to give $-36$ and add up to give $+16$, that is, $\alpha = +18$ and $\beta = -2$.

Split up the middle term into $+18x$ and $-2x$.

$\boxed{3x^2+18x} - \boxed{2x-12}$ Split into two groups

$= 3x(x+6)-2(x+6)$ Factorise each group separately

$= (3x-2)(x+6)$ Factorise again

(ii) Factorise $8k^2+18k-5$

Multiply 8 by $-5$ to give $-40$. Think of two numbers $\alpha$ and $\beta$ which multiply to give $-40$ and add up to give $+18$, that is, $\alpha = +20$ and $\beta = -2$.

Split up the middle term into $+20k$ and $-2k$.

$\boxed{8k^2+20k} - \boxed{2k-5}$ Split into two groups

$= 4k(2k+5)-1(2k+5)$ Factorise each group separately

$= (4k-1)(2k+5)$ Factorise again

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## QUADRATIC FACTORS—METHOD 3

The previous two approaches for factorising might be difficult for some students. A third approach is shown here.

**To factorise** $ax^2+bx+c$**:**

STEP 1 Multiply $a$ and $c$.

STEP 2 Factorise $ac$ so that the sum of the factors $K+L=b$.

STEP 3 Write the expression in factorised form $\dfrac{(ax+K)(ax+L)}{a}$.

STEP 4 Factorise and cancel.

Examples:

(i) Factorise $3x^2+2x-8$

STEP 1 Multiply $3\times-8$, to give $-24$.

STEP 2 The factors of $-24$ are $+6$ and $-4$, since they add to get $+2$.

STEP 3 Write $\dfrac{(3x+6)(3x-4)}{3}$.

STEP 4 $$\frac{(3x+6)(3x-4)}{3}=\frac{3(x+2)(3x-4)}{3}$$
$$=(x+2)(3x-4)$$

(ii) Factorise $6x^2+13x-5$

The two numbers which multiply to give $6\times-5=-30$ and add to give $+13$ are $+15$ and $-2$,

$$\therefore\quad 6x^2+13x-5=\frac{(6x+15)(6x-2)}{6}$$
$$=\frac{3(2x+5)2(3x-1)}{6}$$
$$=\frac{3(2x+5)2(3x-1)}{6}$$
$$=(2x+5)(3x-1)$$

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# APPLICATION OF FACTORISING TO ALGEBRAIC FRACTIONS

Examples:

(i) $\dfrac{3x-6}{4x^3}\times\dfrac{8}{4-2x}=\dfrac{3(x-2)}{4x^3}\times\dfrac{8}{2(2-x)}$ Factorise

$=\dfrac{3(x-2)}{4x^3}\times\dfrac{8}{2(2-x)}$ Note the cancelling with '$-1$'

$=\dfrac{-3}{x^3}$ That is, $\dfrac{x-2}{2-x}=-1$

(ii) Simplify $\dfrac{x^2-x-20}{x^2-25}\div\dfrac{x+1}{x^2+5x}$ Factorise and multiply by reciprocal

$=\dfrac{(x-5)(x+4)}{(x+5)(x-5)}\times\dfrac{x(x+5)}{x+1}$ Cancel where possible

$=\dfrac{x(x+4)}{x+1}$

(iii) Simplify $\dfrac{x^3+8}{x^2-2x-8}$ Factorise $x^3+2^3$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$=\dfrac{(x+2)(x^2-2x+4)}{(x+2)(x-4)}$ Cancel where possible

$=\dfrac{x^2-2x+4}{x-4}$

(iv) Simplify $\dfrac{2x+1}{x-1}-\dfrac{4x-1}{x^2+2x-3}$

$=\dfrac{2x+1}{x-1}-\dfrac{4x-1}{(x-1)(x+3)}$ Factorise

$=\dfrac{(2x+1)(x+3)-(4x-1)}{(x-1)(x+3)}$ Find common denominator

$=\dfrac{2x^2+6x+x+3-4x+1}{(x-1)(x+3)}$ Collect like terms

$=\dfrac{2x^2+3x+4}{(x-1)(x+3)}$

*NOTE* The common denominator could be $(x-1)(x^2+2x-3)$, but this would make the calculation more cumbersome. Once $x^2+2x-3$ is factorised, it is easy to see that the lowest common denominator is $(x-1)(x+3)$.

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## REVIEW EXERCISE – LEVEL 1

**Q. 1** Factorise the following:

**(a)** $9x^2 - 16$

**(b)** $y^3 - 1$

**(c)** $x^2 - 4y^2$

**(d)** $9a^3 - 243$

**(e)** $m^2n^2 - 4p^2$

**(f)** $x^3 + 125$

**Q. 2** Factorise:

**(a)** $xy + 3yz + 4x + 12z$

**(b)** $2ac + 6a + bc + 3b$

**(c)** $p^3 - 2p^2 - p + 2$

**(d)** $6wy + 9wz + 4xy + 6xz$

**Q. 3** Factorise:

**(a)** $2x^2 - 5x - 12$

**(b)** $6x^2 + 5x - 4$

**(c)** $24x^2 - 62x + 35$

**(d)** $20x^2 + 20x - 75$

**(e)** $24x - 16 - 9x^2$

**(f)** $40 + 14x - 12x^2$

**Q. 4** Simplify:

**(a)** $\dfrac{x^3 - 27}{x^2 - 7x + 12}$

**(b)** $x + \dfrac{5x}{x+6}$

**(c)** $\dfrac{5}{27 - 3x} \div \dfrac{9}{18 - 2x}$

**(d)** $\dfrac{x-3}{x+4} \cdot \dfrac{x^2 + 6x + 8}{x^2 + 2x - 15}$

**(e)** $\dfrac{3}{x^2 - 1} - \dfrac{1}{x+1}$

**(f)** $\dfrac{x-1}{1-x} + \dfrac{x-1}{x+1}$

**(g)** $\dfrac{2}{x^2 - x} + \dfrac{3}{x - x^2}$

**(h)** $\dfrac{1}{x^2 - 4} - \dfrac{1}{x^2 + 3x + 2}$

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## REVIEW EXERCISE – LEVEL 2

**Q. 1** Factorise:

**(a)** $24x^2 - 6y^2$

**(b)** $x^4 - y^4$

**(c)** $8 - 27x^3$

**(d)** $x^6 + x^3$

**(e)** $(3x+4)^2 - (2x-3)^2$

**(f)** $x^6 - 26x^3 - 27$

**(g)** $15x + 6xz - 5y - 2yz$

**(h)** $8x^2 - \frac{1}{8}$

**Q. 2** Simplify:

**(a)** $\dfrac{4x^2-y^2}{2x^2+3xy-2y^2} \cdot \dfrac{x^2+2xy}{xy-2x^2}$

**(b)** $\dfrac{x^3+27}{6x^2+21x+9} \div \dfrac{x^2+6x+9}{2x^2+x}$

**(c)** $\dfrac{6xy-x^2-9y^2}{3x^3-24} \div \dfrac{x^2-3x+9y-3xy}{x^2-4x+4}$

**(d)** $\dfrac{x^2-y^2-2x-2y}{x^2-2xy-3y^2} \cdot \dfrac{xy-3y^2}{y-x+2}$

**(e)** $\dfrac{x^2}{4x^2+7x+3} + \dfrac{3x}{4x+3}$

**(f)** $\dfrac{x^2+1}{x^2-1} - \dfrac{x-1}{x^2-2x+1}$

**(g)** $\dfrac{x^3-2x^2}{x^3+8} - \dfrac{x}{x+2}$

**(h)** $x + y + \dfrac{y^2-x^2}{x^2-y^2}$

**Q. 3** Simplify: $\dfrac{2}{2+\sqrt{1-x^2}} + \dfrac{2}{2-\sqrt{1-x^2}}$

**Q. 4** Show that: $\dfrac{(x+1)^{-1} + (x-1)^{-1}}{(x+1)^{-1} - (x-1)^{-1}} = -x$

**Q. 5** Simplify: $\dfrac{9^x + 18^x}{12^x + 24^x}$

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# Chapter 3
# FACTORISING

## Level 1

**Q. 1** (a) $(3x-4)(3x+4)$

(b) $(y-1)(y^2-2y+1)$

(c) $(x-2y)(x+2y)$

(d) $9(a-3)(a^2+3a+9)$

(e) $(mn-2p)(mn+2p)$

(f) $(x+5)(x^2-5x+25)$

**Q. 2** (a) $(y+4)(x+3z)$

(b) $(2a+b)(c+3)$

(c) $(p-2)(p-1)(p+1)$

(d) $(3w+2x)(2y+3z)$

**Q. 3** (a) $(2x+3)(x-4)$

(b) $(3x+4)(2x-1)$

(c) $(6x-5)(4x-7)$

(d) $5(2x+5)(2x-3)$

(e) $(4-3x)(3x-4)$

(f) $2(5-2x)(3x+4)$

**Q. 4** (a) $\dfrac{x^2+3x+9}{x-4}$ (b) $\dfrac{x^2+11x}{x+6}$

(c) $\dfrac{10}{27}$ (d) $\dfrac{x+2}{x+5}$

(e) $\dfrac{4-x}{(x-1)(x+1)}$

(f) $\dfrac{-2}{x+1}$

(g) $\dfrac{1}{x(1-x)}$

(h) $\dfrac{3}{(x-2)(x+2)(x+1)}$

## Level 2

**Q. 1** (a) $6(2x-y)(2x+y)$

(b) $(x-y)(x+y)(x^2+y^2)$

(c) $(2-3x)(9x^2+6x+4)$

(d) $x^3(x+1)(x^2-x+1)$

(e) $(x+7)(5x+1)$

(f) $(x-3)(x^2+3x+9)(x+1)(x^2-x+1)$

(g) $(3x-y)(5+2z)$

(h) $8\left(x-\dfrac{1}{8}\right)\left(x+\dfrac{1}{8}\right)$

**Q. 2** (a) $\dfrac{2x+y}{y-2x}$ (b) $\dfrac{x(x^2-3x+9)}{3(x+3)^2}$

(c) $\dfrac{(3y-x)(x-2)}{3(x^2+2x+4)(x-3)}$ (d) $-y$

(e) $\dfrac{x}{x+1}$ (f) $\dfrac{x}{x+1}$

(g) $\dfrac{4x(x-1)}{(x+2)(x^2-2x+4)}$ (h) $x+y-1$

**Q. 3** $\dfrac{8}{3+x^2}$

**Q. 4** $\dfrac{(x+1)^{-1}+(x-1)^{-1}}{(x+1)^{-1}-(x-1)^{-1}}$

$$= \left[\frac{1}{x+1}+\frac{1}{x-1}\right] \div \left[\frac{1}{x+1}-\frac{1}{x-1}\right]$$

$$= \left[\frac{2x}{(x+1)(x-1)}\right]\left[\frac{(x+1)(x-1)}{-2}\right]$$

$$= -x$$

**Q. 5** $\dfrac{9^x+18^x}{12^x+24^x} = \dfrac{(3\times 3^x)+(6\times 3^x)}{(3\times 4^x)+(6\times 4^x)}$

$$= \frac{3^x(3+6)}{4^x(3+6)} = \left(\frac{3}{4}\right)^x = \frac{3^x}{2^{2x}}$$

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