

Accelerated Maths Learning

EXTENSION 1 (2&3 UNIT COURSE)

UNDERSTANDING YEAR 12 MATHS

SAMPLE

Writing team:

Dr. Max Shur Ph.D. B.Sc.(Hons.)

Erik Hoekstra MA B.Sc. Dip.Ed.

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CHAPTER 1

APPLICATIONS OF GEOMETRICAL PROPERTIES

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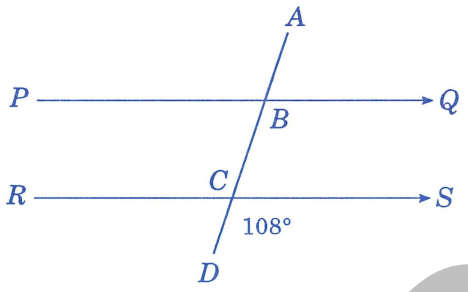
SIMPLE NUMERICAL EXERCISES

You must be able to apply the geometrical properties learned in the Preliminary course to simple theoretical problems requiring one or more steps of reasoning.

Geometrical justifications will be required where appropriate. The best idea is to plan the approach thoroughly, a little like writing an essay: decide where you want to start an what steps will lead you towards your goal. At each step, ask yourself *why* you are able to say what you have said.

Example:



$PQ \parallel RS$

$\angle DCS = 108°$

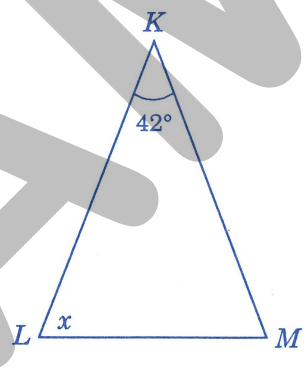
Find $\angle ABQ$.

Solution: In the figure, $PQ \parallel RS$ Given

$\therefore \quad \angle CBQ = 108°$ Corresponding to $\angle DCS$

$\therefore \quad \angle ABQ = 72°$ Supplementary to $\angle CBQ$

Example:



$KM = KL$

$\angle LKM = 42°$

Find $\angle KLM$.

Solution: Since $KM = KL$ ΔKLM is isosceles

$\therefore \quad \angle LMK = x°$ Base angles of isosceles triangles equal

So $42 + x + x = 180°$ Angle sum of a triangle

$2x = 138°$

$x = 69°.$

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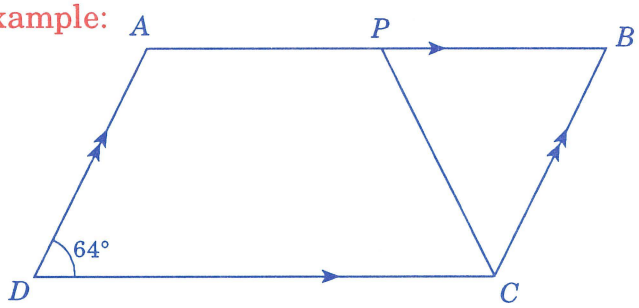
Example:



$ABCD$ is a parallelogram.

P is on AB so that $BC = PC$.

$A\hat{D}C = 64°$

Find $P\hat{C}B$.

Solution: Since $ABCD$ is a parallelogram,

$A\hat{B}C = 64°$ Opposite angles of a parallogram equal

Also, since $BC = PC$, ΔCPB is isosceles.

$\therefore \quad B\hat{P}C = 64°$ Base angles of isosceles triangles equal

So $P\hat{C}B + 64 + 64 = 180°$ Angle sum of a triangle

$\therefore \quad P\hat{C}B = 52°.$

Example: Find the size of the interior angles of a regular octagon.

Solution: The angle sum of an n-sided polygon is $(n-2)\times 180$ [or $(2n-4)$ right angles],

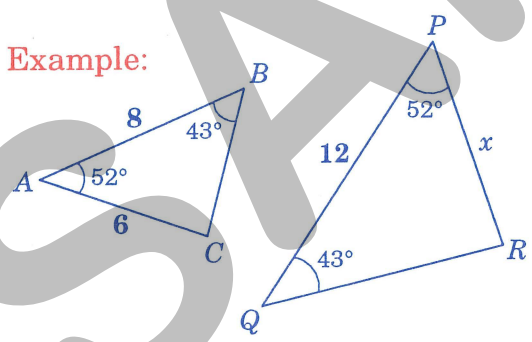
$\therefore$ angle sum of an octagon $= (8-2)\times 180$

$= 1080$

$\therefore$ each angle $= 1080 \div 8$

$= 135°.$

Example:



In the figures, $AB = 8$, $AC = 6$, $\angle BAC = 52°$, $\angle ABC = 43°$; $PQ = 12$, $\angle QPR = 52°$, $\angle PQR = 43°$.

If $PR = x$, find x.

Solution: $\angle BAC = \angle QPR$ Given

$\angle ABC = \angle PQR$ Given

$\therefore \quad \Delta ABC \,|||\, \Delta PQR$

Hence $\dfrac{AB}{PQ} = \dfrac{AC}{PR}$

$\dfrac{8}{12} = \dfrac{6}{x}$

$8x = 72$

$x = 9.$

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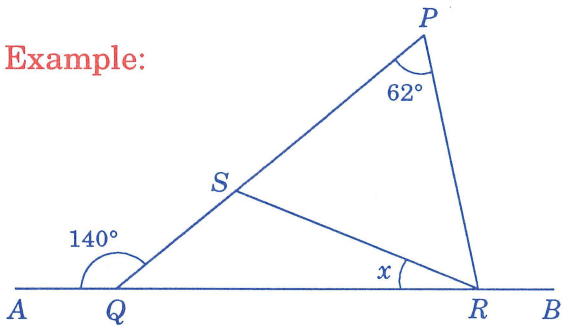
NUMERICAL PROBLEMS
INVOLVING A LARGER NUMBER OF STEPS

These require more planning than those in the previous section. In an exam, you may be 'led' through the steps by having to answer several parts.

Example:



In the diagram, $\angle PQA = 140°$, $\angle QPR = 62°$ and $SP = RP$

Find x.

Solution:

$\angle SQB = 40°$ Supplementary to $\angle AQP$

Since $PR = PS$, ΔSPR is isosceles,

$\therefore \quad \angle PRS = \angle PSR$ Base angles of isosceles triangles

But $\angle PRS + \angle PSR + 62 = 180$ Angle sum of triangle

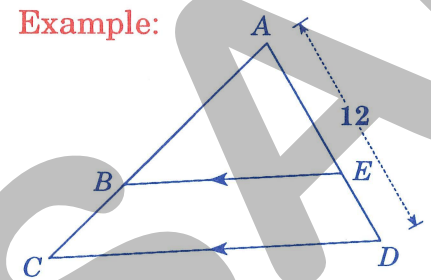
$\therefore \quad 2 \times \angle PSR = 118$

$\angle PSR = 59°$

$\therefore \quad x + 40 = 59$ Exterior angle of a triangle

$\therefore \quad x = 19°.$

Example:



In the diagram, $BE \parallel CD$

$AD = 12$ cm, $AB = 9$ cm,

and $BC = 7$ cm.

(i) Prove that ΔABE is similar to ΔACD.

(ii) Find the length of ED.

(iii) If $CD = 20$ cm, show that $\angle CAD = 90°$.

Solution: (i) $\angle BAE = \angle CAD$ Common angle

$\angle ABE = \angle ACD$ Corresponding angles, $BE \parallel CD$

$\therefore \quad \Delta ABE \parallel\!\parallel\!\parallel \Delta ACD.$

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(ii) $\dfrac{AB}{AC} = \dfrac{AE}{AD} \quad \therefore \quad \dfrac{9}{16} = \dfrac{12-ED}{12}$

$12 \times 9 = 16 \times 12 - 16 \times ED$

$16 \times ED = 84 \quad \Rightarrow \quad ED = 5{\cdot}25 \text{ cm}.$

(iii) $AC^2 + AD^2 = 16^2 + 12^2$

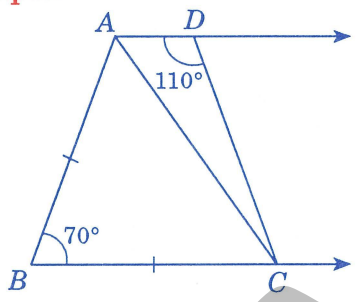
$= 256 + 144$

$= 400; \quad \text{and} \quad CD^2 = 400$

$\therefore \; AC^2 + AD^2 = CD^2$

that is, the triangle obeys Pythagoras' rule, and hence $\angle CAD = 90°$.

Example:



In the figure, $AD \parallel BC$, $AB = BC$, $\angle ABC = 70°$ and $\angle ADC = 110°$.

(i) Show that CA bisects $\angle BAD$.

(ii) Find $\angle ACD$ and $\angle BCD$.

(iii) What special kind of figure is $ABCD$?

Solution: (i) $AB = BC$, so ΔABC is isosceles

$\therefore \quad \angle BAC \times 2 + 70 = 180$ Angle sum of a triangle

$\therefore \quad \angle BAC = 55°.$

Also, $\angle BAD = 110°$ Cointerior to $\angle ABC$, $AD \parallel BC$

$\therefore \quad \angle DAC = 110 - 55$

$= 55°.$

So $\angle DAC = \angle BAC$

$\therefore \quad CA$ bisects $\angle BAD$.

(ii) Now $\angle ACD + 55 + 110 = 180°$ Angle sum of a Δ

$\therefore \quad \angle ACD = 15°.$

Also, $\angle DCB = 70°$ Cointerior to $\angle ADC$

(iii) $ABCD$ is an isosceles trapezium.

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PROBLEMS INVOLVING SIMPLE DEDUCTIONS

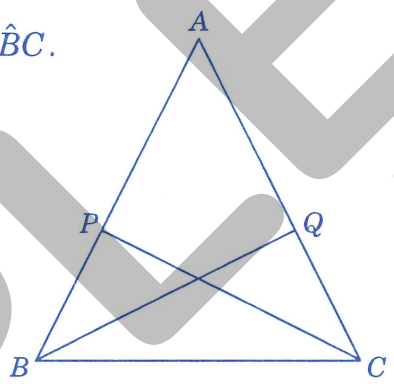
Some problems go beyond asking you to find angles or the lengths of sides; you will need to demonstrate some property of the figure. These are usually the most difficult geometry problems. However, with care and a thorough knowledge of angle and triangle properties, congruency and similarity tests, and the properties of quadrilaterals, you should be able to manage them.

Draw a rough diagram and plan your attack. Make sure each step can be justified by the information given or worked out already.

Example: In the figure, $PC = BQ$ and $P\hat{C}B = Q\hat{B}C$.

(i) Prove that $PB = QC$.

(ii) Prove that $AB = AC$.



Solution: (i)

$PC = BQ$		Given
$P\hat{C}B = Q\hat{B}C$		Given
BC is common		
$\therefore \quad \Delta PBC \equiv \Delta QCB$		(SAS)
and $PB = QC$.		

(ii) Since $\Delta PBC \equiv \Delta QCB$ Above

$P\hat{B}C = Q\hat{C}B$

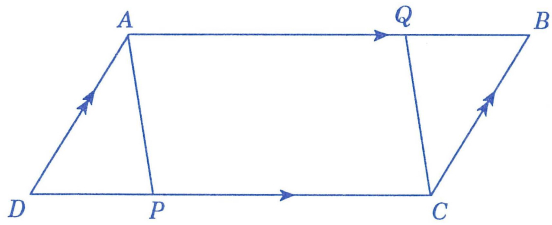
i.e. $A\hat{B}C = A\hat{C}B$

$\therefore$ ΔABC is isosceles and $AB = AC$.

Example: $ABCD$ is a parallelogram and $\angle APD = \angle CQB$.

(i) Prove that $AP = QC$.

(ii) Show that $APCQ$ is a parallelogram.



Solution: (i) Since $ABCD$ is a parallelogram,

$\angle ADC = \angle CBA$		Opposite angles are equal
and $AD = BC$		Opposite sides are equal
Also, $\angle APD = \angle CQB$		Given
$\therefore \quad \Delta APD \equiv \Delta CQB$		(AAS)
and $AP = CQ$.		

SAMPLE

(ii) $\angle APD = \angle CQB$ Given

but $\angle CQB = \angle QCP$ Alternate angles, $AB \parallel CD$

$\therefore \angle APD = \angle QCP$

so $AP \parallel QC$ Corresponding angles are equal

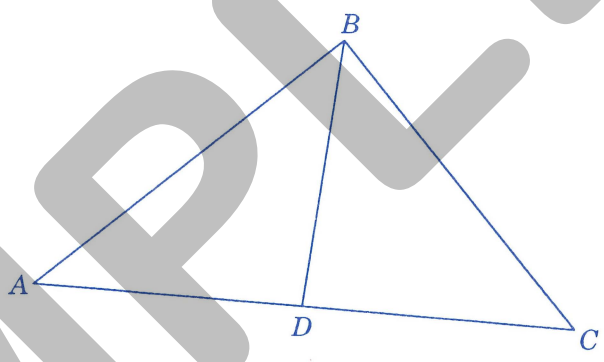
Also $AP = QC$ Above

$\therefore APCQ$ is a parallelogram Opposite sides are equal and parallel

Example: D is the midpoint of the side AC in ΔABC. If D is equidistant from A, B and C, prove that ΔABC is right-angled.

Solution: Draw a triangle ABC and mark D on AC.

Since D is equidistant from A, B and C, $AD = DB = DC$.



Since $AD = DB$, ΔADB is isosceles and $\angle BAD = \angle ABD$.

Also, since $DB = DC$, ΔDBC is isosceles and $\angle BCD = \angle CBD$.

Now $\angle CDB = \angle DAB + \angle ABD$ Exterior angle result

$= 2 \times \angle ABD$ Since $\angle DAB = \angle ABD$

Also, $\angle ADB = \angle DBC + \angle BCD$ Exterior angle result

$= 2 \times \angle CBD$ Since $\angle DBC = \angle BCD$

So $\angle CDB + \angle ADB = 2 \times \angle ABD + 2 \times \angle CBD$

but $\angle CDB + \angle ADB = 180$ ADC is straight

$\therefore 2 \times \angle ABD + 2 \times \angle CBD = 180°$

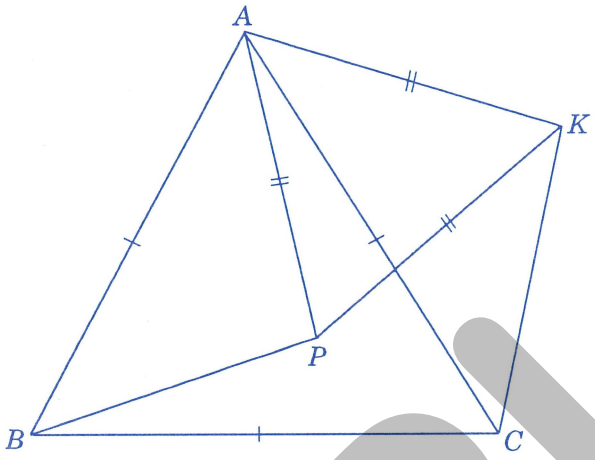
$\angle ABD + \angle CBD = 90°$

i.e. $\angle ABC$ is 90° and ΔABC is right-angled.

SAMPLE

Example: ABC is an equilateral triangle. P is a point inside ΔABC, while K is a point outside ΔABC on the opposite of AC. If ΔAPK is also equilateral, prove that $BP = KC$.

Solution:



All equal sides are clearly marked.

$\angle BAC = 60°$ ΔABC equilateral

$\angle PAK = 60°$ ΔPAK equilateral

$\therefore \quad \angle BAC = \angle PAK$

i.e. $\angle BAP + \angle PAC = \angle PAC + \angle CAK$

$\therefore \quad \angle BAP = \angle CAK$.

Now, in Δs BAP and CAK:

$AB = AC$ ΔABC equilateral

$AP = AK$ ΔPAK equilateral

$\therefore \quad \angle BAP = \angle CAK$ Proved above

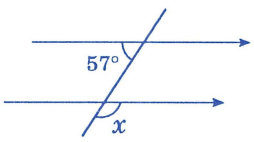
$\therefore \quad \Delta BAP \equiv \Delta CAK$ SAS

$\therefore \quad BP = KC$ Corresponding sides in congruent triangles

SAMPLE

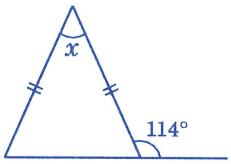
REVIEW EXERCISE

Q. 1 **(a)**



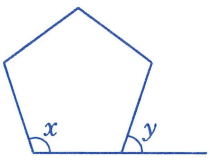
Find x.

(b)



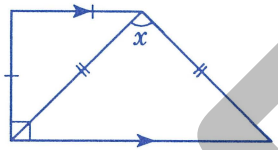
Find x.

Q. 2 **(a)**



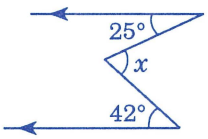
The pentagon is regular.
Find x and y.

(b)



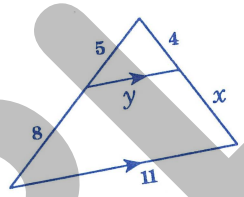
Find x.

Q. 3 **(a)**



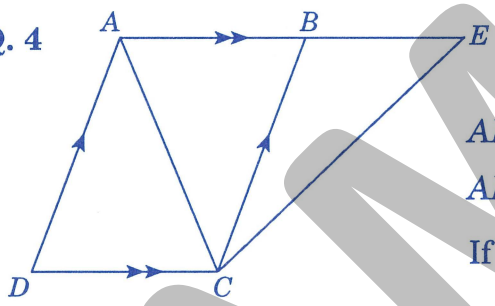
Find x.

(b)



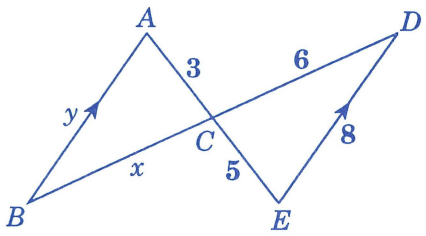
Find x and y.

Q. 4

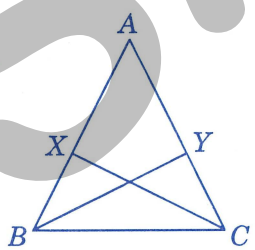


$ABCD$ is a parallelogram. $\angle ACB = 19°$.
AB is produced to E so that $AE = EC$.
If $\angle AEC = 34°$, find $\angle ADC$.

Q. 5 $AB \parallel DE$ Calculate x and y.



Q. 6

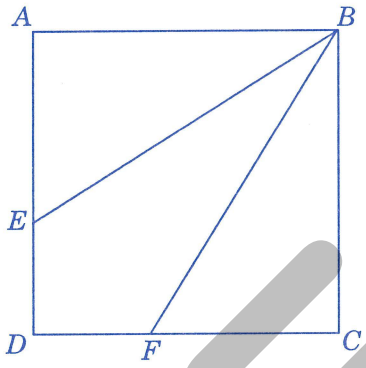


ΔABC is isosceles and $AB = AC$.
X is the midpoint of AB and Y is the midpoint AC.
Prove $\angle XCB = \angle YBC$.

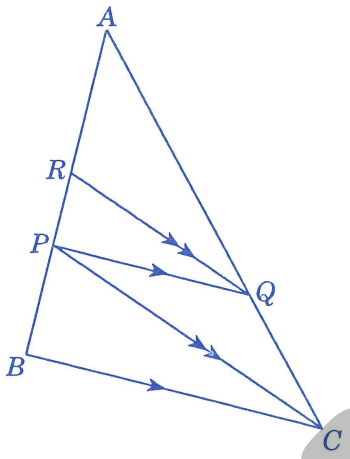
Q. 7 In the parallelogram $ABCD$, X and Y are points on the diagonal AC such that $AX = YC$. Prove that $XBYD$ is a parallelogram.

SAMPLE

Q. 8 $ABCD$ is a square and $BE = BF$.
Prove that $DE = DF$.



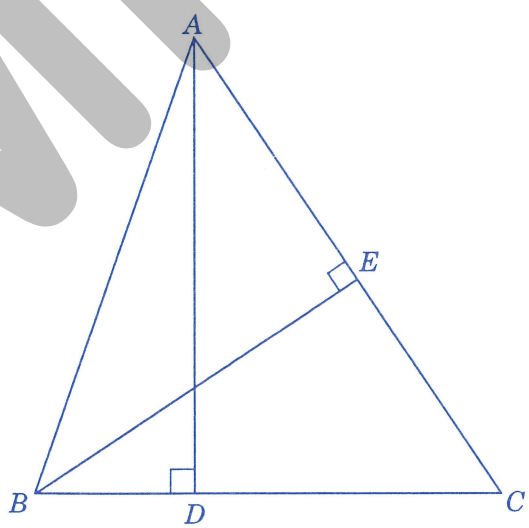
Q. 9 In the figure:



(a) Name two ratios equal to $\frac{AQ}{QC}$.

(b) If $AQ = 8$, $QC = 4$, $PB = 3$, find AP.

Q.10 In the figure:



Prove that $\frac{BE}{AD} = \frac{BC}{AC}$.

Q.11 In a quadrilateral $ABCD$, a line AF parallel to BC meets BD in F and a line BE parallel to AD meets AC in E. Prove that EF is parallel to CD.

Q.12 In a parallelogram $ABCD$, the diagonal BD is perpendicular to AB. If the diagonals intersect at T, prove that $AD^2 = AT^2 + 3TD^2$.

SAMPLE

Chapter 1
APPLICATIONS OF GEOMETRICAL PROPERTIES

Q. 1

(a) $x = 123$

(b) $x = 48$

Q. 2

(a) $x = 108,\ y = 72$

(b) $x = 90°$

Q. 3

(a) $x = 67°$

(b) $x = 6{\cdot}4,\ y = 4{\cdot}23$

Q. 4 88°

Q. 5 $x = 3{\cdot}6,\ y = 4{\cdot}8$

Q. 6 In ΔBCX and ΔBCY:

(i) $XB = YC$ Midpoints of isosceles triangle

(ii) $A\hat{B}C = Y\hat{C}B$ Equal base angles of isosceles Δ

(iii) $BC = BC$ Common side

$\therefore\ \Delta BCX \equiv \Delta BCY$ **SAS**

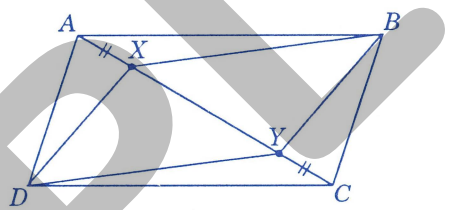
$\therefore\ \angle XCB = \angle YBC$

Q. 7 Firstly, prove that: $\Delta AXD \equiv \Delta BYC$ **SAS**

$\therefore\ XD = BY$

Secondly, prove that: $\Delta AXB \equiv \Delta DYC$ **SAS**

$\therefore\ XB = DY$



Since the opposite sides are equal, the figure must be a parallelogram

Q. 9 (a) $\frac{AP}{PB}, \frac{AR}{RP}$ (b) $AP = 6$

Chapter 2
COORDINATE METHODS IN GEOMETRY

Q. 1 Show

Q. 2

(a) $AB = \sqrt{26}$

$AC = \sqrt{130}$

$BC = \sqrt{104}$

(b) ΔABC is right angled

(c) $A = 26$

Q. 3

(b) Diagonals bisect

Q. 4

(b) (3, 1) and $\left(\frac{3}{2}, 0\right)$

(c) kite

Q. 5

(a) $2x - 3y + 1 = 0$

(c) isosceles

Q. 6 $(-3, 4)$

SAMPLE